UTILITY FUNCTIONS WITH JUMP DISCONTINUITIES: SOME EVIDENCE AND IMPLICATIONS FROM PEASANT AGRICULTURE

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For many empirical studies it is appropriate to allow explicitly for a disaster avoidance motive. This is one of the objectives of the use of safety-first models that use some variant of minimizing the probability of disaster or maximizing return given a constraint on the probability of disaster (Roy, 1952; Boussard and Petit, 1967). It may also be useful to define disaster avoidance using an expected utility model where there is a jump or vertical section (and a consequent non-concavity) in the utility function. The jump represents a large disutility associated with the loss of another dollar. This type of utility function has some interesting portfolio implications. For example, an individual may invest proportionately more of his portfolio in a project as the variance on the project's return increases. Disaster avoidance is likely to be exhibited where capital markets are inefficient, as may be true in some areas of peasant farming. The empirical relevance of this type of utility function is here demonstrated by the use of data on peasant farming in Mexico.

I. THE THEORY

The concept of disaster avoidance is not new in economic theory. Several closely related topics—safety-first, focus-loss and chance-constrained programming—are all based on this notion. In one of the earlier safety-first models, Roy (1952) hypothesizes that there may be some critical level of income below which an individual faces death or bankruptcy and uses the criterion that an individual will minimize the probability of falling below this income level. He recognizes that the same results may flow from expected utility maximization and a very simple form of a utility function with a jump discontinuity. But the function he discusses lacks intuitive appeal. It is based on a discrete utility function with a value of one when the firm is not bankrupt and a value of zero when it is bankrupt (Roy, 1952; p. 433).[1] In his model there is no increasing utility for higher in-

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1. Lintner (1965; p. 19) points out that with normality Roy's results may be derived from a more general utility function.

comes when the probability of bankruptcy is zero, and from this flows his simple minimization-of-disaster criterion.

A more plausible formulation might be a utility function that is a positive monotonic function of income but exhibits a jump discontinuity at some critical income level (see Figure 1).[2] This jump in the utility function could arise from a preference for one state of the world (e.g., not being bankrupt) over another or from a jump in the earnings function due to a discrete change in earning potential associated with a one-time loss that drops income below some critical level.

UTILITY FUNCTION OF INCOME WITH A JUMP DISCONTINUITY
(Zero income may be to the right of D)

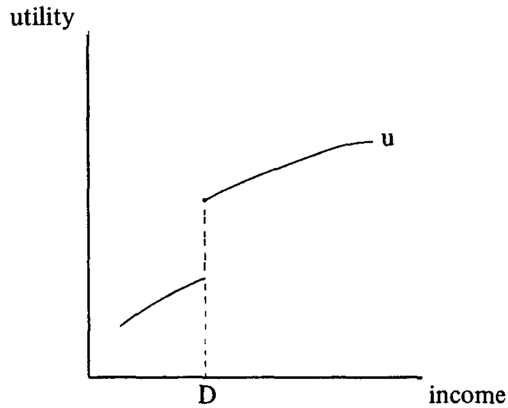


Figure 1

One could view this derived or indirect utility function as coming from an analysis of a person's multiperiod decision problem. The individual may obtain utility from income and the state of the world. Declaration of bankruptcy could then reduce utility by either or both of the two factors. There could be a direct utility effect due to a dispreference for bankruptcy or an indirect utility effect from a discontinuity in future earnings capacity due to declaring bankruptcy. Either or both of these factors could then yield a jump discontinuity in the *derived* utility function. (This derived utility function could be a recursion equation in a dynamic programming formulation of the problem.)

2. The utility function may be flat or positively sloped to the left of this critical income level. If bankruptcy laws or death results in an individual's not caring about how large a loss he incurs, as long as it is greater than the critical loss, then that portion of the utility function is horizontal. If greater losses still imply greater decreases in utility then it is positively sloped.

Various conditions other than bankruptcy could also give rise to a similar jump discontinuity. Where credit rationing occurs there may be a critical income level below which one will lose his firm, farm, bullock, etc. or his home, furniture, etc. (A wife's threat to leave could play a similar role.) This asset loss could lead to a direct utility loss due to embarrassment about repossession, or to an indirect utility loss due to a reduction in earnings capacity as in the case of losing a bullock from a farm.[3] Job tenure considerations can also yield such a discontinuity. A salesman could lose his job if his sales were too low or an executive his job if his firm's performance falls too far. These cases can yield a similar jump in the utility function if there is a critical earnings level below which the employee will not be retained. Again the utility loss can be direct (e.g., loss of respect, etc.) or indirect (e.g., from being branded a failure by potential employers). Any of these factors, if associated with a critical income level, may cause a jump in *the function that relates an individual's utility to his income level.*[4]

II. EMPIRICAL EVIDENCE OF UTILITY FUNCTIONS WITH JUMP DISCONTINUITIES

We have found some direct estimates of utility functions with discontinuities and near-vertical sections consistent with our hypothesis. The data are reported in O'Mara (1971) from his study of diffusion of technical change in and around a farm project in Mexico. He interviewed 72 farmers in order to assess their risk preferences. He used their answers to a series of hypothetical gambles to form their utility functions for changes in current income. Each utility function is constructed so as to have zero utility for a zero payoff and 10 utiles for an additional 10 units of money, each unit being 1000 pesos. We may then compare subsections of a utility function with its average slope in this range. By construction the average slope of the right-hand side of each utility function is one. Each utility function was assessed by finding income levels which, if given with certainty, would yield -10, -7.5, -5, -2.5, 0, 2.5, 5, 7.5, and 10 utiles, and then a piecewise approximation was made over this range. For the data used see O'Mara (1971; pp. 302-342).

3. In the absence of a single-interest-rate capital market one may be forced to sell a unique asset (e.g., a house or an adjoining acre of land) at a price at which he would in the future gladly repurchase the asset. But unless he extracted all of the buyer's consumer surplus or the buyer has changed his mind he may have to pay (much) more for it.

4. If the probability of disaster rises very rapidly over a very small income range then given the derived nature of this function the utility function will be very steep, but not discontinuous. If the probability rises over a very wide range the utility function will, other things being equal, be less steep. In some cases the probability may make a discrete jump at a given income level. It is with those cases that we are primarily concerned here although a very rapid increase over a narrow range is the same general phenomenon and gives qualitatively similar results.

The disaster avoidance model would predict jump discontinuities in the utility functions at or to the left of the zero point, i.e., for income losses. Discontinuities to the right of the zero point should be rare because these imply that even to remain at the current income level would be disastrous. Most people with discontinuities to the right of the zero point would in fact not remain in farming and thus would not show up in the sample. There are at least 7 out of 72 (9.7%) discontinuities (i.e., sections that look vertical), and these are all to the left of the origin. By use of the binomial distribution, the null hypothesis that jumps are equally likely to the left and the right of the origin may be rejected at the 99 per cent level. But there may be even more discontinuities in the utility functions. Even with perfect measurement, a true utility function with a "vertical section" of less than 2.5 utiles would not, by this piecewise approximation, yield a measured discontinuity. It would yield a very steep approximation to this section. In 29 (40%) of the cases, part of the slope to the left of the zero point is 25 or more times the average slope of each individual's function between zero and 10,000 pesos.[5] Nineteen of these had slopes greater than 50 and eleven had slopes over 500. These all tend to support the hypothesis. Also in examining the 29 cases where there were slopes of 25 or more we find that 6 of these have a portion, to the left of the very steep or discontinuous portion, that flattens all the way back to a slope of 1 or less, and 20 of them flatten out to slopes of 10 or less. This shows definite evidence of a non-concavity of a sort that might imply a shift to strong *risk-loving* behavior in last-ditch attempts if disaster were imminent.

There are, however, some steep sections of these curves to the right of the origin. But of these there are only 4 with slopes over 25, 3 with slopes over 50, and none with slopes over 500. A null hypothesis that there is an equal probability of slopes of 25 or more to the left or at the zero point and to the right of the zero point may be rejected at the 99 per cent level ($t = 4.35$).[6]

Another possible interpretation of these discontinuities is that they are due to measurement error. Indeed, in two cases reported as discontinuities above there are measured slopes of the wrong sign. In absolute values their slopes are in one case over 50 and in the other about 10. It seems unlikely,

5. Of course these may simply be observations of a very steep range of the utility function. This may be because the probability of disaster increases rapidly in some small income range. Then qualitatively and functionally this would be a different cut from the same cloth.

6. The test used was a "zero test." The null hypothesis is that there is an equal probability of slopes of 25 or more to the left and right of the origin. The test uses the binomial distribution and the normal approximation yields the t ratio. The t ratio for slopes of 50 or more is also significant at the 99 per cent level ($t = 3.41$) and the probability level (calculated directly) for slopes of 500 or more is .9985.

however, that two consecutive points could be mismeasured with exactly the right error to show a discontinuity if there were none.[7] Therefore, as a cross check for mismeasured points we examined the measured utility functions over ranges of two measured points, i.e., 5 utiles or more. To the left of the origin we find 20 and 6 cases of slopes of 25 and 500 or more, respectively, and 4 which have discontinuities over this range. There are no comparable sections to the right of the origin.[8]

Since steep sections are rare to the right of the origin but common to the left of the origin these tests support the hypothesis that discontinuities (or sections very close to vertical) are likely to exist and that these are associated with economic losses as in a disaster avoidance model.

III. SOME PORTFOLIO IMPLICATIONS IN PEASANT AGRICULTURE

This type of utility function may induce portfolio behavior that seems counterintuitive. A risk-averse investor may, due to disaster avoidance, shift his portfolio toward more investment in one prospect as the variance on its return increases. Of course, such a phenomenon may exist even with a continuously differentiable risk-averse utility function, but it may be induced by a disaster avoidance motive in cases where it would not otherwise exist.

This will be demonstrated by an example.[9] The subsistence farmer is likely to have, in addition to a disaster avoidance motive, a more or less fixed total investment (e.g., fixed land). Assume he is faced by a decision between only two investments—a high-mean-high-variance choice versus a low-mean-low-variance choice, such as a cash crop versus a subsistence crop, e.g., jute and rice in Bangladesh (Hussain 1969).

Consider a farmer who has no disaster-avoidance motive and who is diversifying between the high-mean-high-variance crop (jute) and the low-mean-low-variance crop (rice). Suppose he is planting half of his land in

7. Those individuals answering questions about hypothetical gambles showed this result. In the original research design O'Mara intended to "verify" the hypothetical functions by observing cropping decisions between two cropping techniques. In fact, in assessing these farmers' priors he in all but three cases found first-degree stochastic dominance which meant that cropping decisions were independent of the forms of the utility functions so there is no behavioral verification possible. But his results on cropping decisions are consistent with interview assessments of priors, which yield some greater confidence in these estimated utility functions. Cf. O'Mara (1971).

8. Testing whether slopes of greater than 25 or greater than 500 over a range of 5 utiles are equally common on either side of the origin yields rejections at the 99 and 98 per cent level, respectively. A similar test for jumps of 5 utiles or more yields only a 90 per cent significance level.

9. Although the existence of a perfectly safe asset, or the possibility of unlimited borrowing at a single interest rate, could change the implications presented here (cf. Lintner (1965)), this is probably not relevant in the case of peasant farming. Such a safe asset, if one existed, might yield a safe return which is lower than the disaster level of income. In this case its existence would not affect the results. And unlimited borrowing at a single interest rate is even less likely to occur in this case than elsewhere.

rice and half in jute. Next assume that each of these crops has a finite lower bound on the probability distribution on returns and the lower bound of the jute distribution is below that for rice. (See Figures 2a and 2b). Now assume that the spread of returns on rice increases without the mean return changing and that as the variance on rice increases the farmer with no disaster avoidance motive would shift to less rice and more jute production. For simplicity we assume that these two returns are independently distributed and that both crops are grown with constant returns to scale.

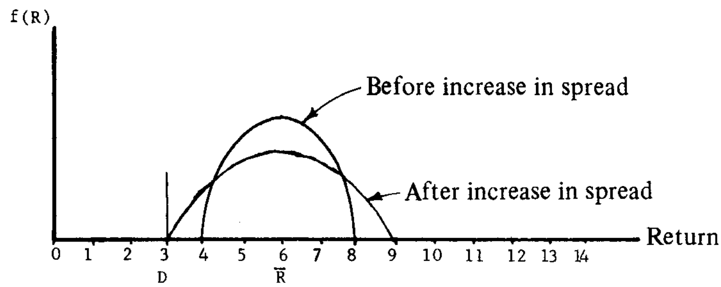
Now suppose instead that someone tells this farmer (e.g., his landlord or his wife) that if his return isn't above D dollars in the next time period something drastic will happen. Let us further assume that at least D dollars is certain to be earned by an allocation of land half to rice and half to jute production at the low variance level on rice return *but* that an allocation of half rice and half jute will not assure D dollars of return if the variance on rice rises to the new level (see Figure 2c). *If the disaster is considered to be serious enough* (i.e., the vertical gap in the utility function is sufficiently great), when the variance on rice return increases the farmer will increase rice production to maintain a zero probability of falling below D dollars of return. For the distributions shown in Figure 2, if the spread of the rice function increased to cover the range (3, 9) as shown, then the optimal solution is to specialize in rice. (If the spread increased considerably beyond this, however, the farmer would plant only jute). Even though his mean return falls as he shifts more heavily into rice, he is able to assure himself of not falling below D dollars of return. Thus the introduction of a disaster level of income may reverse some portfolio decisions in some ranges.[10] In such a case it is not at all clear that an increase in the variance of return on a low-mean-low-variance crop should lead to a decline in investment on that crop and this should be taken into consideration when estimating agriculture surplus functions. Indeed Roumasset (1971; pp. 19, 26, 28) shows, using a safety-first model, that the possibility of a shift into a crop as its variance goes up may be relevant for the study of Philippine agriculture.

Our model has other portfolio implications also. Consider an agricultural development agency that is considering the introduction of one of two new higher (mean) return crops (or techniques) to an agricultural area. Further assume that they must choose between one or the other

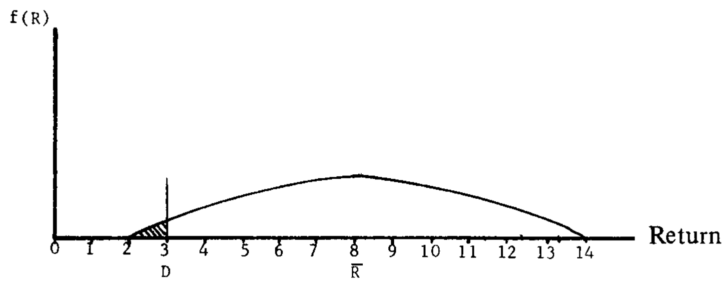
10. A general proof for distributions with finite lower bounds is available from the author. Neither the finite lower bound nor the jump is a necessary condition for this phenomenon to exist. The result of shifting into the production of a product as the variance on its return increases need not rely on this jump nor on this specific form of density function, but rather the jump can introduce such behavior where it would not otherwise exist; cf. Rothschild and Stiglitz (1971).

PROBABILITY DENSITY FUNCTIONS FOR DIFFERENT RICE-JUTE COMBINATIONS
($\bar{R}$ equals mean return, D is a disaster level of return)

2a Specialized in rice



2b Specialized in jute



2c Half rice-half jute

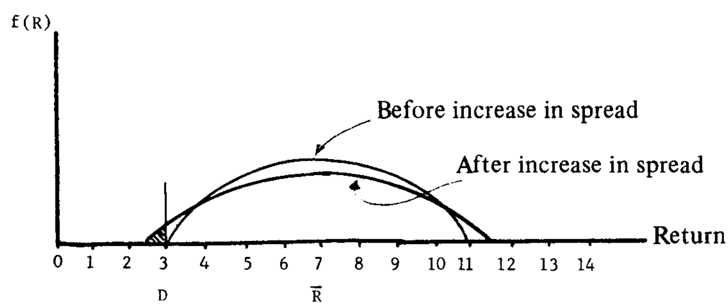


Figure 2

crop, e.g., two types of hybrid corn. If one crop has a slightly higher mean than the original crop but the same lower bound on returns, it may be more readily adopted than a crop with a much higher mean return but with an inferior lower bound on returns even if the variance on returns is not higher in the latter case. The appropriate choice in this case may be to introduce the lower return crop or to assure an adequate lower bound on return through an appropriately selected support price, direct subsidies, and/or crop insurance for those who adopt the new technology.[11]

IV. CONCLUSION

In this paper it is hypothesized that in some circumstances one might expect a disaster avoidance motive to introduce a jump in a decision maker's utility function. One area in which this is likely to occur is agricultural development. This paper presents empirical evidence supporting the likelihood of this occurring in peasant farming and shows that this should be taken into account for estimating agricultural surplus functions and deciding upon an appropriate development policy. As a general proposition this would imply that in cases where disaster avoidance is more likely to be a primary motive (e.g., subsistence farming or illiquid small business) this motive and its portfolio implications should be explicitly recognized for analysis.

11. Cf. a recent paper written for economists and agronomists by Jock Anderson (1974).

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