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OLIGOPOLISTIC PRODUCT WITHHOLDING IN RICARDIAN MARKETS

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ABSTRACT

We consider strategic behaviour in the rental market for quality-differentiated goods. In his classic analysis Ricardo showed that at the competitive equilibrium the price of the marginal unit is driven to zero. An oligopolistic market structure usually leads to a radically different equilibrium. Deliberate withholding of units often becomes part of a firm's best response, and whenever this occurs, a pure strategy equilibrium fails to exist. A necessary but not sufficient condition for a pure strategy equilibrium to exist is for one firm to own all the best quality units. A mixed strategy equilibrium always exists and the associated payoff is always greater than the competitive payoff.

INTRODUCTION

In Ricardo's classic analysis of rent, when the firms (or rentiers) are competitive, the best unused unit determines the equilibrium rent generated by the superior quality of the units which are used.[1] The assumption of competition is critical for the Ricardian analysis. Many markets in which payoffs are scarcity rents do not exhibit competitiveness. Examples

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[1] In the words of Ricardo, 'When in the progress of society, land of the second degree of fertility is taken into cultivation, rent immediately commences on that of the first quality, and the amount of that rent will depend on the difference in the quality of these two portions of land.' See Ricardo (1917).

include the markets for television reruns, hotel accommodations, airline seats, and rental real estate.[2]

Even in a very simple model, dropping the competitive assumption can lead to a radical alteration in the equilibrium. Using a Bertrand (Nash in prices) solution concept with duopolistic firms, we show that a pure strategy equilibrium (PSE) usually does not exist, though a mixed strategy equilibrium (MSE) always does. We show that a necessary (but not sufficient) condition for the existence of a PSE is that one firm owns all the best units and that the unique PSE occurs at the Ricardian prices. Further, at any MSE, firms' expected payoff exceeds the Ricardian payoff. All results extend to the case of oligopoly. [See Masson *et al.* (1987), Mudambi (1986) and ICF Inc. (1983).]

The nature of equilibrium with price-setting firms has been widely studied in the literature.[3] We analyse the specific case where each unit of the good is unique.

THE MODEL

We treat the product as an 'input', e.g., TV reruns, blocks of hotel rooms or agricultural land. The market is analysed as a rental market.[4] We assume the rental price of the product to be non-negative. The quality of the product is assumed to be quantifiable in terms of its generation of revenue. 'q_i' is the quality level of the ith unit of the product where $i=1$ denotes the highest quality unit, $i=2$, the second highest, etc. $R(q)$ is the per period revenue-generating capability from quality q (gross of rental price). There are assumed to be a large number of buyers. The number of units available is denoted by 'N' and the number demanded by 'n'.[5]

THE COMPETITIVE CASE

In the simple competitive case, ownership of the good is completely dispersed. Each firm has only one unit of the good to sell. The components

[2] The aspect of rental markets which makes them perfect illustrations of Ricardian analysis is the total perishability of the product. An unsold airline seat, an empty hotel room or apartment, all represent revenue lost forever.

[3] See for example Dixon (1984), Maskin (1986) and Allen and Hellwig (1986) among others. Our results are similar in spirit to those obtained by Kreps and Scheinkman (1983) for the homogeneous product case.

[4] By using the capitalized value of the net revenue streams instead, the analysis could just as easily be carried out in terms of final sales. If the product is a final good, $R(\cdot)$ represents the net pecuniary equivalent of the (per period) utility of consumption.

[5] This demand formulation is more general than the normal one, for which demand would be positive if $R(q)>0$. That is, n may be restricted by another scarce factor, e.g., number of farmers available or numbers of available 'time slots' for airing a television program.

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of the Ricardian competitive equilibrium price vector P^R, are

$$p_i^R = \begin{cases} R[q_i] & \text{if } n \geq N. \\ R[q_i] - R[q_{n+1}], \quad \text{for } i < n+1 & \\ 0, \quad \text{otherwise} & \text{if } n < N. \end{cases} \qquad (1)$$

The case of excess demand ($n \geq N$) and total rent extraction is the limiting case; the equilibrium remains the same under varying market structures. The more interesting case is that of excess supply. Therefore we assume $n < N$ in what follows.

ILLUSTRATIVE EXAMPLES

Before proceeding to the general analysis of the duopoly case, we provide three examples to illustrate the non-existence of a Nash PSE and the lower bound of the support of a MSE.

Example 1

There are a total of six units ($N = 6$) and four are demanded ($n = 4$). The revenue-generating capacity of the units is the vector [100, 90, 80, 70, 60, 50]. If the ownership of the units is completely dispersed, the price of the 5th unit is competed to zero. The Ricardian equilibrium price vector [40, 30, 20, 10, 0, 0] prevails.

Let firm 1 own the best, third best and fifth best units with firm 2 owning the remaining three units. If either firm charges the Ricardian prices ([40, 20, 0] for firm 1; [30, 10, 0] for firm 2) it can sell its best two units in the face of any non-negative prices chosen by its rival. Hence any price vector involving lower prices cannot be an equilibrium.

If either firm charges prices higher than the Ricardian prices, the best response of its rival is to undercut these prices and divert sales to itself. Thus, any price vector higher than the Ricardian price vector cannot be an equilibrium if all units are offered for sale.

If firm 1 withholds its third best unit and charges 50 and 30 for its best two units, it can sell these units in the face of any non-negative prices offered by firm 2. This strategy yields a certain payoff of 80 which is greater than the Ricardian payoff of 60. Thus, the Ricardian prices are not firm 1's best response to any play by firm 2, and cannot be an equilibrium.

Firm 2 knows that firm 1 will not accept a payoff of less than 80. Thus, firm 2 can add $(2/3) \times 10 = 6.667$ to the price of its best two units. This yields a payoff of $36.667 + 16.667 = 52.334$. Firm 1 has no incentive to undercut these prices, since by doing so, and selling all its units, it gets marginally less than 80, i.e., $(46.667 - \varepsilon) + (26.667 - \varepsilon) + (6.667 - \varepsilon) = 80 - 3\varepsilon$.

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Thus, at any MSE, firm 1's payoff is at least 80 and firm 2's payoff is at least 52.334. These compare with the Ricardian payoffs of 60 and 40 respectively.

Example 2

Consider the same scenario as in Example 1, but re-arrange the ownership pattern, so that firm 1 owns the best four units and firm 2 owns the remaining two. Knowing that the lowest price firm 2 can charge is zero, firm 1's best initial move is to withhold the third and fourth best units and charge the revenue-generating capacity for the best two units, yielding a payoff of 190. Firm 2's best response is to charge 60 and 50 for its two units and obtain 110. Bertrand price shading follows and a PSE fails to exist. Thus, even if one firm owns all the best units, a PSE may not exist.

Example 3

Now consider the same scenario as in Example 2, i.e., firm 1 owns the best four units and firm 2 owns the remaining two, but let the revenue-generating capacities be [100, 90, 80, 70, 20, 10]. Firm 1's optimal strategy here is to withhold no units and charge the Ricardian prices, obtaining a payoff of 260. Firm 2's best response is to charge zero and we have a PSE at the Ricardian prices.

It may be seen from the above examples that even when one firm owns the best units, the critical factor determining whether a PSE exists or not is the concentration of ownership and quality levels of the marginal units. For instance, in comparing examples 2 and 3, it is seen that by altering the quality levels of the marginal units, we move from a situation with no PSE to one with a PSE.

THE DUOPOLY CASE

For the duopoly we denote the number of units owned by the firms as N_1 and N_2 ($N_1 + N_2 = N$). The ownership pattern is the result of a process of assigning N ranked units to 2 firms. Therefore, an *outcome* is defined to be a set of product quality levels (one for each unit) combined with an ownership pattern.[6] Let an arbitrary outcome be denoted by Q and the set of all outcomes by S.

We must describe not only the quality ranking of all units from 1 to N, but also the mapping between the best, second best,... unit of firm j and

[6] We can think of any single pattern of quality levels of the units as a draw from an N-dimensional probability distribution, i.e., the actual state of the world is a draw from the continuum of all possible states of the world.

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the global ranking. For firm j, define $r_j(1),..,r_j(N_j)$, as j's mapping from its best through its N_jth unit into the global ranking. If $r_j(k)=L$, j's kth best unit is globally the Lth best unit available.

Denote the quality and price of the units owned by firm j by $Q_j=\{q_{r_j(1)},..,q_{r_j(N_j)}\}$ and $P_j=\{p_{r_j(1)},..,p_{r_j(N_j)}\}$. Define $P\equiv\{P_1,P_2\}$; P_j is firm j's choice vector. Define P_j^R to be the value of P_j whose components take the Ricardian values (1). Let $Q\equiv\{Q_1,Q_2\}$. Let $d_{r_j(k)}(P,Q)$ be an indicator function with the value one if firm j's kth best unit is sold and the value zero if it is not. The value depends on the price and quality vectors. Then j's pay-off function is

$$U_j(P)=\sum_{k=1}^{N_j} d_{r_j(k)}(P,Q)\,p_{r_j(k)}, \quad j=1,2 \tag{2}$$

Denoting firm j's price space by Φ_j, our game is $[\Phi_j, U_j \mid Q; j=1,2]$.

If a firm removes one of it units from the market, this unit is said to be *withheld*. In a withholding strategy, firm j offers fewer than N_j units. Rather than varying the dimension of Φ_j, we assume that firm j withholds its kth best unit by setting $p_{r_j(k)}>R[q_{r_j(k)}]$; i.e., by setting a price greater than the unit's revenue-generating capacity.[7]

In order to analyse this game, it is convenient to partition the outcome space into mutually exclusive and collectively exhaustive sub-sets. The optimising behaviour of firms can then be characterised in each of these sub-sets, denoted E_1 and E_2.

E_1 is the set of all outcomes where the global best $n+z$ units ($z\geq 0$) are owned by one firm. Formally,

$$E_1=\{Q\in S \mid r_j(1)>n \text{ for some } j=1,2\} \tag{3}$$

E_2 is the complement of E_1. Then $E_1\cup E_2=S$.

Proposition 1

There does not exist a pure strategy equilibrium (PSE) for any outcome in E_2. ■

If both firms own units among the best n, a PSE does not exist.

Proof

No price vector containing a price $p_i<p_i^R$ can be an equilibrium as both firms can obtain the Ricardian prices with certainty. No price vector

[7] Since the good in question may be durable, withholding by the firms is credible to buyers only if (a) fairly stringent contractual arrangements are made with regard to the withheld units (b) the buyers either have a relatively high rate of time discount or vary in their valuation of the good, so that all of them do not wait in anticipation of the withheld units eventually appearing on the market. See Coase (1972), and the subsequent literature on durable goods monopoly [e.g. Bulow (1982), Kahn (1986), Gul *et al.* (1986) etc.].

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containing a price $p_i > p_i^R$ can be an equilibrium; Bertrand best responses will ensure price shading.

Consider the price vector P^R itself. Let firm 1 own the global $n+1$st best unit and firm 2, the $n+2$nd. P^R is not an equilibrium because Firm 1's best response to P_2^R is to withhold the $n+1$st unit and charge prices to preclude sale of the $n+2$nd best unit.

Proposition 2

For outcomes in E_1, there exists a pure strategy equilibrium (PSE) if and only if

$$\sum_{h=0}^{n-k-1} R(q_{n-k-h}) - R(q_{n+k+1}), \quad 0 \leq k \leq n-1 \tag{4}$$

is maximized at $k=0$.[8]

If one firm owns the best n units, a PSE exists if and only if this firm never finds withholding optimal.

Proof

(a) Let firm 1 own the best n units. Suppose (4) is maximised for $k=0$. From (4), its best response to P_2^R $(=0)$ set by firm 2 is to set the Ricardian prices P_1^R. If firm 1 sets prices at P_1^R, firm 2's best response is to set P_2^R. We have a PSE at the Ricardian prices (1).

(b) Suppose (4) is maximized for $k>0$. Now firm 1's best response to P_2^R $(=0)$ set by firm 2 is to withhold units with global ranks $n-(k-1)$ to n. Successive best responses involve further withholding and it follows that a PSE fails to exist. ■

The game possesses a PSE only over a subset of E_1. Further, the PSE involving the Ricardian prices is the only PSE.

THE EXISTENCE OF A MIXED STRATEGY EQUILIBRIUM

The main problem in raising the question of a MSE is that the payoff functions of the firms are not continuous. It is well-known that in a Bertrand game, the payoff functions are discontinuous where the players' strategies coincide. [See for example Chamberlin (1956), Beckmann (1967) and Levitan and Shubik (1972).]

This problem is dealt with in Dasgupta and Maskin (1986), who prove the existence of a MSE for several classes of games with payoff function discontinuities. Below we state one of their theorems as Proposition 3.

[8] Proposition 2 as stated and proved relates only to those outcomes in E_1 where $z=0$, i.e., the firm owning the best n units does not own the $n+1$st unit. When $z>0$, the units with global ranks $n+1$ to $n+z$ will always be withheld in any pure strategy best response. Thus, in a discussion of pure strategy equilibria, they may be ignored.

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Proposition 3

Suppose that there are 2 agents and that for $j=1,2$ $U_j:\Phi_j \to \mathbb{R}$, is continuous and bounded except on the subset $\{(P_1, P_2) | P_1 = P_2\}$. If it is true that

$$\lim_{P_1 \overset{-}{\to} P,\ P_2 \overset{+}{\to} P} U_1(P_1, P_2) \geq U_1(P, P)$$

$$\geq \lim_{P_1 \overset{+}{\to} P,\ P_2 \overset{-}{\to} P} U_1(P_1, P_2) \qquad (5a)$$

$$\lim_{P_1 \overset{-}{\to} P,\ P_2 \overset{+}{\to} P} U_2(P_1, P_2) \leq U_2(P, P)$$

$$\leq \lim_{P_1 \overset{+}{\to} P,\ P_2 \overset{-}{\to} P} U_2(P_1, P_2) \qquad (5b)$$

and the left (right) inequality in (5a) is strict if and only if the right (left) inequality in (5b) is strict; then the game $[\Phi_j, U_j; j=1,2]$ has a MSE.

Subject to the pecuniary value of the quality differential, Proposition 3 proves the existence of a MSE for precisely the type of payoff discontinuities that we encounter.

As an illustration of this point, consider example 1. The Ricardian prices $[P_1^R = (40, 20, 0); P_2^R = (30, 10, 0)]$ are a point of payoff discontinuity, at which the payoffs are $[U_1 = 60;\ U_2 = 40]$. For $P_1 = (40-\varepsilon,\ 20-\varepsilon,\ -\varepsilon) \overset{-}{\to} P_1^R$, $P_2 = (30+\varepsilon,\ 10+\varepsilon,\ \varepsilon) \overset{+}{\to} P_2^R$, the payoffs are $[U_1 = 60 - 3\varepsilon;\ U_2 = 30 + \varepsilon]$. This is a case where the left inequality in (5b) is strict. If we consider $P_1 = (40+\varepsilon,\ 20+\varepsilon,\ \varepsilon) \overset{+}{\to} P_1^R$, $P_2 = (30-\varepsilon,\ 10-\varepsilon,\ -\varepsilon) \overset{-}{\to} P_2^R$, the payoffs are $[U_1 = 40 + \varepsilon;\ U_2 = 40 - 3\varepsilon]$, and the right inequality in (5a) is strict. It can be verified that the conditions for Proposition 3 are always met. We conclude that our game always possesses a MSE.

COMPARING THE EQUILIBRIA

We now compare the prices at the duopoly equilibrium with the Ricardian equilibrium prices (1). We only consider outcomes which generate MSE, as it is for these outcomes that the equilibrium is sensitive to market structure. We therefore examine the support of the duopoly equilibrium.

It is clear that no price vector which yields less than the Ricardian payoff can be in the support of a MSE, since this payoff may be obtained with certainty. However, a stronger lower bound on the minimum acceptable payoff may be obtained.

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Proposition 4

For all outcomes in E_2 and for outcomes in E_1 where (4) is maximized at $k>0$, the lower bound of the payoffs for both firms, over prices in the support of a MSE is strictly greater than the Ricardian competitive payoff.

For all outcomes where a PSE fails to exist, the firms' payoffs exceed the Ricardian competitive payoff with certainty.

Proof

Let firm 1 own the global $n+1$st unit. Denote the number of firm 1's units that are among the best n by t_1, i.e., $r_1(t_1)\leq n$, $r_1(t_1+1)=n+1$. Denote the pure strategy of withholding the global $n+1$st unit and setting prices to just preclude the sale of the global $n+2$nd unit by W_1. The W_1 payoff is strictly greater than the Ricardian payoff and is available with certainty, i.e., regardless of the strategy adopted by firm 2. Any firm 1 strategy in the support of a MSE must yield a payoff at least as great as the W_1 payoff.

As firm 1 will accept no payoff lower than the W_1 payoff,

$$[t_1/(t_1+1)][R(q_{n+1})-R(q_{n+2})]$$

can be added by firm 2 to the Ricardian price of each unit which it owns among the best n. Denote this pure strategy by W_2. *This is not the Nash best response to* W_1. It is merely a strategy which yields a strictly greater payoff than the Ricardian payoff with certainty. Any firm 2 strategy in the support of a MSE must yield a payoff at least as great as the W_2 payoff.

The proof also illustrates the factors determining the excess of the lower bound over the Ricardian payoff. In particular it illustrates that as the ownership of the marginal units becomes concentrated, the lower bounds for the payoffs obtained by the firms rise. Interestingly, increasing the ownership concentration of used units, *ceteris paribus*, has no effect on the lower bound.

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