

## Oligopoly in Advertiser-Supported Media

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*In this article, advertiser-supported media, such as television, are analyzed as an industry selling audiences to advertisers. A simple stylized model is used to demonstrate that increased competition leads to less of a price decline (in extreme cases, maybe even a price increase) than would be expected in other industries. This arises because audience diversion introduces terms similar to conjectural variations in equilibrium output. Further, in this model, it is shown that if greater competition makes advertisers better off, it makes the media-consumers worse off and vice versa. The extension to mixed subscriber-advertiser supported media is demonstrated to lead to similar, albeit attenuated, conclusions.*

Most media tend to be suppliers of advertising time or space. As private corporate entities, firms in such industries can be expected to act as profit-maximizers. However, due to the inherent impact of the media on the public interest, the issue of competition in such industries is judged to be extremely important by legislators and regulators. Yet the manner in which competition affects the media is only partially understood. This article examines one important aspect of market structure, the role of competition in influencing the price of advertising.

Advertising density can reduce the demand for a television show or a magazine. We demonstrate that if some lost audience goes to rivals, this may significantly alter the results of competition. This is because lost audiences are not lost customers, but "factors of production," in the market for selling audiences to advertisers. This diversion effect is shown to lead to output below Cournot levels and the possibility for advertising prices to rise with increased competition.

For expository ease we focus the discussion on television broadcasts, although it would equally apply to radio and some other media. Broadcast television is "free" to viewers. Others have analyzed the effects of additional competition on the "diversity" of programming available to television viewers (for example, [5, 6, 9, and 10]). But little has been said about how additional competition affects the sales of advertising. For broadcast tele-

vision the only true "market" is the one for sales of audiences to advertisers. And for many other media, this market provides the bulk of total firm revenues, with revenues from product sales to readers or viewers providing relatively little.

We demonstrate that viewer diversion effects act like an additional conjectural variation term, and this significantly alters oligopoly equilibria. Earlier work examined the gains to viewers through increased diversity. In the market for advertising time, the interests of advertisers and viewers are at loggerheads. Reducing advertising density generally pleases viewers, but raises prices to advertisers. So if competition does lower advertising prices, it has done so through more viewer advertising contacts.

The results of the article (albeit in attenuated form) generalize to priced (often print) media, subject to the restriction that the media product not be demanded by final consumers solely for its advertising content. The analysis of the case of priced media is presented in the Appendix. The one substantial difference between the cases is that when the medium's product has a positive subscription price, the interests of the advertisers and the consumers (readers or viewers) are not as directly opposed.<sup>1</sup>

#### MODELING

The model concentrates on advertising and abstracts from program quality and other means of increasing audiences. For analytical ease, we assume that there is only one price for advertising (per viewer-minute) in the industry.<sup>2</sup> The individual maximization decisions of the advertisers generate an industry demand function for television advertising. The equilibrium price will be a function of the total number of viewer-minutes supplied by all sellers.

In allocating time to view television, each viewer optimizes over the available options for spending leisure time. We assume that members of the audience feel "the fewer ads the better," which implies the optimal time allocation for television viewing declines as the number of advertising minutes rises. Aggregating such decisions over all viewers yields a total audience response function. This would be dependent upon various parameters characterizing viewer choice, including program quality. However, within the framework of a partial model, the audience response function may be set up as a function of just the vector of advertising minutes chosen by the firms in the industry.

Firms in actual practice have considerable latitude in setting the quantity of advertising minutes. This is sufficiently important that some sports league contract negotiations are on the permissible advertising breaks. Avid sports fans will notice that some breaks in the action are filled with "analysis" rather than ads, or that some ad breaks overlap play. The Leagues wish to enhance the present value of future viewership, whereas advertisers and programmers can more readily switch to alternative outlets.

Further, "stripped" re-runs or old movies are often accompanied by varying quantities of advertising minutes. ("Stripping" is the practice of playing several seasons of an old weekly series on a nightly basis.)<sup>3</sup> Avid viewers, especially of movies, will note where some program deletions also affect the plot lines. Similarly, on radio, considerable efforts are made by stations to tout their low allotment of time to advertising (for example, "More Music Radio," or playing "five in a row with no commercial interruptions"). Xerox used to advertise its television Specials with no commercial interruptions.

### Two Examples

To see the driving force of the model in its starkest form, we present two polar examples. Although both use the same equilibrium concept, one is analogous to the familiar Cournot mineral spring problem, while the other adjusts like a Bertrand model and reaches an opposing result.

Suppose there are two firms, each with 0 costs. For the first example each has a fixed and immutable audience size indexed to unity. Further, let the demand function for television advertising be linear and of the form  $p = a - bV$ , where  $p$  is the price of advertising per viewer-minute, and  $V$  is the total industry supply of viewer-minutes. In this example,  $V = m_1 + m_2$ , where  $m_i$  is the number of ad-minutes supplied by firm  $i$ . Firm  $i$  faces the problem of maximizing  $\pi_i^c = [a - b(m_1 + m_2)]m_i$  with respect to  $m_i$ . The superscript "c" indicates the Cournot duopoly. The Nash equilibrium to this game is clearly the same as that to the familiar, strictly analogous Cournot mineral spring problem;

$$(1) \quad m_i^* = a/3b, \text{ for } i = 1, 2; \text{ and } p^* = a/3.$$

To model the same market as a monopoly, two scenarios are possible. The monopoly could control one outlet with an audience of 2 (the sum of the audiences of the duopolists) or it could control two outlets, each with a fixed audience equal to unity. Since the pricing results are identical for the two problems, we present only one of them, the two station monopoly. The monopolist seeks to maximize  $\pi^m = [a - b(m_1 + m_2)](m_1 + m_2)$  with respect to  $m_1$  and  $m_2$ . Here "m" denotes a monopoly. The symmetric solution is<sup>4</sup>

$$(2) \quad m_1^{**} = m_2^{**} = a/4b; \text{ and } p^{**} = a/2.$$

Given this audience structure a monopolist provides fewer viewer-minutes of advertising to advertisers, driving up the price of advertising. The outputs and prices are identical to the merger of two Cournot mineral springs. Further, with the monopoly, the viewer will see fewer minutes of advertising per program. Each viewer faces a greater number of advertising minutes under duopoly than under monopoly.<sup>5</sup>

This result is, however, a polar case. It relies on the assumption that

audience sizes are insensitive to the number of advertising minutes broadcast by each firm. Dropping this assumption leads to systematic changes in the equilibrium.

To see this, we structure a second polar example. Assume a different audience reaction function, but suppose that the model is otherwise the same. Viewers derive disutility from any program with more than  $a/5b$  minutes of advertisements. (They derive no disutility from  $a/5b$  minutes of advertising "intermissions" in the program.) Further, they otherwise regard all programs as having identical utility and are equally spread across firms if they are indifferent between them.

Now in the duopoly case, the equilibrating process that evolves is analogous to a homogenous product Bertrand price game. Maintaining the assumption that their total audience (the sum of the audiences of the two firms) is fixed at 2, the implied audience reaction functions are

$$(3) \quad A'(m_1, m_2) = \begin{cases} 2, & \text{if } m_j > m_i \text{ and } m_j > a/5b \\ 1, & \text{if } a/5b > m_i \text{ and } a/5b > m_j \\ & \text{or if } m_i = m_j \\ 0, & \text{if } m_i > m_j \text{ and } m_i > a/5b \end{cases}$$

$i \neq j, i, j = 1, 2.$

Each firm's objective function (using "b" to highlight the Bertrand analogy) is

$$(4) \quad \pi_i^* = [a - b(A^1 m_1 + A^2 m_2)] A'(m_1, m_2) m_i, \quad i = 1, 2,$$

which must be maximized with respect to  $m_i$ . The only Nash equilibrium to this game is

$$(5) \quad m_i^{**} = a/5b, \text{ for } i = 1, 2; \text{ and } p^{**} = 3a/5b.$$

Each firm would provide no more than  $a/5b$  minutes of advertisements (for fear of losing the entire audience) and no less (since profits could be increased). The cut-off number of advertising minutes,  $a/5b$ , plays a role analogous to marginal cost in the simple, constant-returns-to-scale, Bertrand price game.

Now consider the case of a monopoly. Its symmetric equilibrium output of advertising minutes is unaffected by the introduction of the audience response function.<sup>6</sup> This is because (3) only determines how the total audience (assumed constant) is allocated among firms. The equilibrium remains (2) with price,  $p^{**} = a/2$ .

Unlike the first example, in the second the duopoly offers fewer equilibrium advertising viewer-minutes to advertisers than does a monopoly. Increasing competition leads to a higher price of advertising with the duopoly than with the monopoly. At the same time, each viewer faces fewer  $[a/5b]$  advertising minutes per program in the duopoly case than with the monopoly cases.<sup>7</sup> These are polar examples, but they reflect a



more general principle. That is, if the equilibrium level of advertising causes viewers to divert to rivals at the margin, then the price effects of increasing competition will be attenuated relative to "normal" markets. This is because the outlets may wish to compete for advertisers by increasing their own advertising viewer-minutes, but in the process they may increase rivals' viewer minutes. As will be demonstrated in the next section, it is this diversion to rivals, rather than simply the diversion of viewers from watching, that drives the results.

#### A More General Model

Let there be  $N$  firms in the industry. Clearly, viewers have  $N + 1$  options — they may view the program of one of the firms or not view at all. If all firms have positive audiences, we refer to this as an "interior" point. As a firm increases its number of advertising minutes, more viewers move to viewing the programs of other firms or not viewing at all. If at least some viewers choose to switch to the programs of other firms, then increasing the number of ad minutes by a firm increases the audience sizes of its rivals. Thus, for interior points, the audience response function of firm " $i$ " is

$$(6) \quad A^i = A^i(m_1, m_2, \dots, m_N), \quad A_i^i < 0, \quad A_j^i > 0, \\ j \neq i, \quad i = 1, 2, \dots, N.$$

The firm's audience response function is somewhat analogous to a production function with externalities. The firm sells viewer-minutes to its advertisers. A firm's increase in advertising minutes acts like a "negative" factor of production on the audience component of its output. Its rivals' ad minutes act as a "positive" one. We shall assume that each firm's audience response function is twice continuously differentiable, strictly concave, and uniformly bounded over all interior points.<sup>8</sup> Further, we will assume that it attains a global maximum and minimum at a finite point in its domain.<sup>9</sup>

The total audience for television is the sum of the audiences of the individual firms. Given that viewers dislike ads, an increase in advertising minutes by a firm should decrease total audience, as some viewers tune out rather than switch channels. Thus, total audience will be decreasing in minutes broadcast by any of the firms. For interior values we may write

$$(7) \quad A = A(m_1, m_2, \dots, m_N) = \sum_{i=1}^N A^i(m_1, m_2, \dots, m_N), \quad A_i < 0, \\ \text{for all } i = 1, \dots, N.$$

Further, in the interior, the total audience function, as the sum of strictly

concave functions, is itself strictly concave. From (6) and (7), it has the property

$$(8) \quad 0 > A_i = \sum_{j=1}^N A_i^j > A_i^i, \quad i = 1, 2, \dots, N.$$

In other words, as firm  $i$  increases its minutes, total audience does not fall as rapidly as  $i$ 's audience, as some viewers are diverted to the programs of other firms.

#### The N-Firm Nash Equilibrium

We assume that the industry demand curve for advertising is similar to that used in the examples, and of the form

$$(9) \quad p = a - b \left[ \sum_{j=1}^N A^j m_j \right],$$

where  $p$  is the industry price of advertising per viewer-minute.

The firms are assumed to play Nash best responses in choosing their minutes of advertising. To simplify the notation we denote  $A^i m_i$ , the output of the  $i^{\text{th}}$  firm in viewer-minutes, as its quantity of viewer-minutes,  $q^i$ . Using (9) and (6), the problem of the  $i^{\text{th}}$  firm is to maximize

$$(10) \quad R^i = \left( a - b \left[ \sum_{j=1}^N q^j \right] \right) q^i, \quad i = 1, 2, \dots, N,$$

with respect to  $m_i$ . The implicit reaction function is

$$(11) \quad q_i = \left[ a - b \sum_{j \neq i} q_j^* \right] / \left\{ b \left[ q_i^* + \sum_{j=1}^N q_j^* \right] \right\}, \quad i = 1, 2, \dots, N,$$

where  $q_j^*$  is the derivative of  $q^j$  with respect to  $m_i$ . From (11) we may obtain implicit expressions for the  $N$ -firm symmetric Nash equilibrium quantity and price. These are

$$(12a) \quad q^*(N) = a / \{ b[N + Q_1(N)] \}$$

$$(12b) \quad p^*(N) = a Q_1(N) / [N + Q_1(N)],$$

where

$$(13) \quad Q_1(N) = \sum_{j=1}^N (q_j^* / q_i^*)$$

and all derivatives are evaluated at  $q^*(N)$ .

The symmetry assumption is made purely for notational convenience. The nonsymmetric Nash equilibrium has virtually identical properties, though it is not as clearly manipulated. For an explicit treatment of the nonsymmetric Nash equilibrium, see [4].

Since viewers are an input, and not paying customers, their tendency to switch from firms that increase their advertising minutes to *alternative viewing* introduces a term equivalent to a nonzero conjectural variation into (11) and (12). Were it possible for  $q_i^j$  to equal zero for all  $j \neq i$ , then (12a) and (12b) would reduce to the usual Cournot-Nash equilibrium values of  $a/(N+1)b$ , and  $a/(N+1)$  respectively.

It is immediate from (8) that  $q_i^i > 0$ . If  $R^i$  is concave in  $m_i$ , then it can be shown that  $q_i^i > 0$  at any interior Nash equilibrium.<sup>10</sup> This means that  $Q_i(N) > 1$  and the Nash equilibrium output (12a) will be lower than the Cournot-Nash output level and the Nash equilibrium price (12b) will be correspondingly higher. Audience diversion to rivals causes the equilibrium output of viewer-minutes to be restricted.

These qualitative results can be stated in another form. If viewers are not diverted to other advertiser-supported firms when advertising minutes increase, then the price of  $a/(N+1)$  results even if viewers are diverted from watching television. Thus, if rival programming is seen as a poor substitute by discouraged viewers of one firm (even if turning off the set appears to be a good substitute), then the Cournot-Nash price results. It is the "production externality" by which increased advertising leads some viewers to switch to rivals' programs that causes the Nash equilibrium price to exceed the Cournot-Nash price.

It now becomes clear that this phenomenon is not driven by the demand specification.<sup>11</sup> The result is not simply a "quality" result that discourages viewers from watching one outlet, but one that flows from the effect of viewers being diverted to rivals. In any demand specification where increasing minutes passively increases rivals' "output" [leads indirectly to an advertising price decrease through increases in rivals' audiences] through diversion, the qualitative effects of this model survive.

### Comparisons

We now examine the effects of competition on the equilibrium (12). There are two ways in which competition may be reduced in this model. The first is to reduce the number of firms and broadcast facilities. The second is to reduce the number of firms, but not the number of broadcast facilities, by increasing the number of facilities owned by each firm.

In the first case each firm always controls one broadcast outlet. We compare the equilibrium output and price when there are  $N$  firms with that when there are  $(N-1)$  firms.

This comparison is not straightforward because the audience functions have domains of different dimensions. We deal with this problem by assuming that the audience function when there are  $(N-1)$  firms is the same as when there are  $N$  firms, but with the advertising minutes on the  $N^{\text{th}}$  channel (which is now off-the-air) set high enough that audience for that channel would be 0 for all minutes choices of the  $(N-1)$  existing

firms.<sup>12</sup> This assumption solves the dimensionality problem associated with comparing equilibria across different levels of competition. Further, it incorporates "gains to diversity." From (6),  $N$  stations with minutes of  $(m^o, \dots, m^o)$  leads to a greater total audience (but a lower audience per station) than would be the case were there  $(N - 1)$  stations with unaltered advertising minutes. [That is, if minutes were  $(m^o, \dots, m^o, \bar{m}_N)$ , where  $\bar{m}_N$  is high enough that  $A^* = 0$ .]

Using (12) and (13), the symmetric Nash equilibrium quantity and price for  $(N - 1)$  firms, each with one broadcast outlet, are

$$(14a) \quad q^*(N - 1) = a / \{b[(N - 1) + Q_1(N - 1)]\}$$

$$(14b) \quad p^*(N - 1) = a Q_1(N - 1) / [(N - 1) + Q_1(N - 1)].$$

From (12) and (14), it follows that a unit increase in the number of firms leads to a higher equilibrium price if and only if

$$(15) \quad (N - 1)/N > Q_1(N - 1)/Q_1(N).$$

*Increasing Competition May Raise or Lower Prices*

Now we consider the case where competition is lessened through a reduction in the number of firms with the number of broadcast outlets fixed at  $N$ . Since we are considering symmetric equilibria, we reduce the number of firms from  $N$  to  $N/2$ , increasing the number of outlets owned by each firm to two. We retain the audience functions as defined in (6) and (7). The  $i^{\text{th}}$  firm owns the  $i^{\text{th}}$  and  $k^{\text{th}}$  outlet and  $k = (N/2) + i$ . Its profit is

$$(16) \quad R^{iT} = \left\{ a - b \left[ \sum_{j=1}^N q^j \right] \right\} (q^i + q^k), \quad i = 1, 2, \dots, N/2.$$

Maximizing (16) with respect to  $m_i$  and  $m_k$  leads to the implicit reaction functions

$$(17a) \quad q^i + q^k = \left\{ \left[ a - b \sum_{j \neq i, k} q^j \right] (q^i + q^k) \right\} / \left\{ b \left[ q^i + q^k + \sum_{j=1}^N q^j \right] \right\}$$

$$(17b) \quad q^i + q^k = \left\{ \left[ a - b \sum_{j \neq i, k} q^j \right] (q^i + q^k) \right\} / \left\{ b \left[ q^i + q^k + \sum_{j=1}^N q^j \right] \right\},$$

$$i = 1, 2, \dots, N/2.$$

Assuming that the  $N$  outlets are all identical, the solution will be at a

symmetric point,  $q^{*0} = q^{*0}$ . Solving (17), a symmetric Nash equilibrium (in implicit form) may be expressed as

$$(18a) \quad q^0(N/2) = a/b[N + Q_2(N)],$$

$$(18b) \quad p^0(N/2) = aQ_2(N)/[N + Q_2(N)],$$

where

$$(19) \quad Q_2(N) = \left( 2 \sum_{j=1}^N q_j^i \right) / (q_i^i + q_i^i),$$

and all derivatives are evaluated at  $q^0(N/2)$ . From (12) and (18), we conclude that increased competition leads to a higher price in equilibrium if and only if

$$(20) \quad Q_1(N) > Q_2(N).$$

*Again, Prices May Rise or Fall*

Our analysis of the effects of competition on advertising price can be brought out most sharply by comparing a monopoly to a duopoly. In this comparison, with  $i = 1$ , both (15) and (20) reduce to  $q_1^2 > q_1^1$ , since  $Q_1(1) = 1$ ,  $Q_2(2) = 2$  and  $Q_1(2) = 1 + [q_1^2/q_1^1]$ . This highlights a salient feature of advertiser-supported media. Prices will rise with increased competition if the effect of competition on audience diversion  $q_j^i (j \neq i)$ , is large relative to its direct effects on audience loss,  $q_i^i$ .<sup>13</sup>

The intuition for why raising  $N$  may raise prices can be most clearly seen in this monopoly/duopoly example. A monopoly need not worry about losing audience to rivals. But if viewers are very likely to switch to a firm with lower advertising minutes, duopolists may end up pushing ad-minutes towards 0 as they compete to gain audience *size* to sell to advertisers.

Also, in equilibrium  $q_i^i > 0$ .<sup>14</sup> So, if the number of firms is altered without changing the number of broadcast stations (as in (18)), the effects on total viewer-minutes and minutes-per-station have the same sign. So with respect to advertising, the interests of viewers and of advertisers are directly opposed. If increased competition makes advertisers better off, it makes viewers worse off and vice versa. Lower ad prices mean more ads per program.<sup>15</sup>

The intuition behind (15) and (20) can be seen in the profit function. Ad-minutes can raise a firm's profits by raising its viewer-minutes sold. But ad-minutes can reduce profits by depressing the advertising price. As the firm increases *ad-minutes* its own *viewer-minutes* ("quantity") rise (near equilibrium) and its  $N$  rivals' viewer-minutes also rise through audience diversion. For any given level of  $N$ , the market is less "competitive" than the corresponding Cournot-Nash analogue due to these diversion effects. Phrased differently, increasing competition from a monopoly to  $N$  firms will have a less competitive price impact for these markets. (Or merger to

monopoly will have a less deleterious competitive effect for these markets.) Theoretically, the diversion effect may be strong enough to cause rising prices with increasing competition!

### CONCLUSION

The model provides a link between advertising time as having value to advertisers and having disutility for viewers. Firms in the broadcast media like television stations, and especially radio stations, individually stress their restricted numbers of advertisements to viewers/listeners, and this model demonstrates how this competition for audiences affects competition in advertising.

The model is potentially relevant for assessing numerous shifts in the entertainment and advertising industries. Currently the US is seeing the entry of Fox Broadcasting, a fourth television network. We are also observing an expansion of pay-cable channels, where viewers pay to view programs without ads (or as many ads). (The case of combined subscription-advertising media is covered in the Appendix.<sup>16</sup>) Also, the government has recently eased the antitrust rules against merging rival newspapers and is considering removal of the prohibitions against cross-media ownership in any geographic market. Each of these should be analyzed in light of the possibility that diversion effects may reduce the competitive consequences of structural changes on *advertising markets* but keeping in mind the potential tension created when more advertising is seen as "good" by advertisers, but "bad" by viewers. Of course the *degree* to which the competition-attenuating effect has an impact is an empirical question, and its effects almost certainly will be affected by changes in programming as well as advertisements. But it is a question not heretofore even asked, and it should be asked when making policy.

Finally, the potential application of the insights in the model extends beyond these situations. Advertisements are now increasingly being broadcast at theaters, on airline flights, and interspersed in the Muszak in department stores. And structural change in advertising markets is evolving rapidly, and differently, in other countries. Italy has virtually atomistic television advertising competition; England is now letting it in; and the new European satellite system will present an even larger variety of interests.

### APPENDIX

We now extend the model to allow for a positive product price. We analyze the first case in the text, that is, lessening competition through reducing the number of firms and outlets together.

We maintain all other assumptions in the text. Each firm now has two choice variables — its level of advertising units (minutes in broadcasting, space in printed media) and its subscription price. The audience functions are functions of the vectors of advertising minutes and of subscription prices. The audience function for the  $i^{\text{th}}$  firm is

$$(A.1) \quad A^i = A^i(m_1, m_2, \dots, m_N, r_1, r_2, \dots, r_N), \quad i = 1, 2, \dots, N.$$

In (A.1),  $r_i$  is the price net of (constant) marginal production costs. The price  $r_i$  is indexed such that one unit of audience is a unit of product sold. So quantity of product is  $A^i$ , whereas quantity of advertising is  $q^i = m_i A^i$ . We assume that (6) is the same as (A.1) with the subscription price vector set to 0. The  $(N-1)$  firm audience function is the same as (A.1) with the minutes-subscription price combination on the blank channel set to reduce its audience to 0. We denote the partial of  $A^i$  with respect to the  $j^{\text{th}}$  firm's ad minutes by  $A^i_j$  as before, and the partials for  $A^i$  and  $q^i$  with respect to the  $j^{\text{th}}$  subscription price as  $A^i_{r_j}$  and  $q^i_{r_j}$  respectively. We would expect higher prices to reduce audiences. That is,

$$(A.2) \quad A^i_{r_i} < 0, A^i_{r_j} > 0, \quad i \neq j, i, j = 1, 2, \dots, N.$$

Since, audience size now equals quantity of product sold, profits from product sales are

$$(A.3) \quad S^i = r_i A^i, \quad i = 1, 2, \dots, N.$$

Using (10), total profits are

$$(A.4) \quad T^i = R^i + S^i, \quad i = 1, 2, \dots, N.$$

The first order conditions for a maximum are

$$(A.5a) \quad T_i = -bq^i \sum_{j=1}^N q^j_i + pq^i_i + r_i A^i_i = 0,$$

$$(A.5b) \quad T_n = -bq^i \sum_{j=1}^N q^j_n + pq^i_n + A^i + r_i A^i_n = 0.$$

Define  $(A^i_i/A^i_n) = z_i$ . Substituting for  $r_i$  in (A.5a) from (A.5b) and noting that  $q^i_i - z_i q^i_n = A^i$ , firm  $i$ 's implicit reaction function is

$$(A.6) \quad q^i = [a - z_i - b \sum_{j=1}^N q^j_i][q^i_i - z_i A^i_n]/b[q^i_i - z_i q^i_n + \sum_{j=1}^N q^j_i - z_i \sum_{j=1}^N q^j_n].$$

From (A.6) the symmetric Nash equilibrium in implicit form is

$$(A.7) \quad q^{**}(N) = \{a - z(N)\}/\{b[N + Q(N)]\},$$

where all derivatives are evaluated at  $q^{**}(N)$ , dependence on  $N$  is explicitly introduced,  $z = z_i$  for all  $i$ , and

$$(A.8) \quad Q(N) = \sum_{j=1}^N \{[q^j_i - z(N)q^j_n]/[q^i_i - z_i(N)q^i_n]\}.$$

The equilibrium with  $(N-1)$  firms can be similarly derived. Advertising quantity supplied will fall (and the ad-price rise) as  $N$  rises if  $Nq^{**}(N) < (N-1)q^{**}(N-1)$ . A sufficient condition for this to occur is

$$(A.9) \quad z(N)/z(N-1) > \{1 + [Q(N)/N]\}/\{1 + [Q(N-1)/(N-1)]\} > 1.$$

The second inequality may be written as  $(N-1)/N > Q(N-1)/Q(N)$ , which is similar to (15). The intuition is the same as that provided in the article since the numerator of  $Q(N)$  captures the effect on audience diversion and the denominator, the effect on audience loss.

The first inequality tells us that for price to rise with  $N$ , the marginal audience loss through ad minutes must be strong relative to the marginal loss through raising the subscription price. In other words, viewers must be more intolerant of increased ad minutes relative to increased subscription price.

A modified version of (A.9) to take account of multi-outlet ownership is sufficient to ensure that the same result holds with the reduction of competition through fewer firms and a constant number of broadcast outlets.

We now consider the relationship between  $q^*(N)$  and  $q^{**}(N)$ . Note that  $S_i$  is concave in  $m_i$  since  $A_i$  is concave in  $m_i$ . Thus, if  $R_i$  is concave in  $m_i$ ,  $T_i$  is concave in  $m_i$  from (A.4). Since  $S_i < 0$  for all values of  $m_i$ ,  $T_i$  evaluated at  $q^*(N)$  is negative. Thus, the optimal choice of advertising minutes is smaller with a positive subscription price (because minutes divert subscription revenues). We have shown in note 9 that at the value of  $m_i$ , which maximizes  $R_i$  (and for smaller values),  $q_i$  is positive. This implies that  $q^{**}(N) < q^*(N)$ .

For a given  $N$ , advertisers are worse off if the media can charge a positive subscription price because the quantity (of viewer-minutes) supplied falls and price rises. Viewers are now subjected to fewer minutes of advertising per program (or ads per issue) but pay for this through the positive subscription price. Thus, the interests of advertisers and viewers are no longer directly opposed. (Of course the model remains partial; with free and easy entry, subscription revenues would lead to greater  $N$ .)

#### NOTES

\* This research started when we were examining the effects of the Financial Interest and Syndication Rule on television program prices, a topic we do not address here, but do for one scenario in [3].

1. It should be noted that there are some media for which the demand may be increasing in the number of advertisements. For these, the results of the model are, of course, reversed.

2. We model demand as a function of the quantity of viewer-minutes (ad-viewer contacts) sold. Ads are priced by (expected) audience times minutes of ad time, but there are other factors affecting demand, such as demographics and whether additional views of a single ad have diminishing returns on "purchase intent." (See [8] for how firms view the demand for advertising.) Along related lines, in [1], it is shown how ad repetition effects may endogenously affect the level of the demand curve as the number of stations changes. Although several (including Rust) feel that the effects of these valuation formulas are overly stressed in practice, Rust notes that they are what practitioners use. Briefly, increased numbers of channels fragment the audience. This leads to a lower value of weekly viewership to an advertiser, as some viewers see an ad "too many" times. At the same time, for any aggregate television audience size, it raises the marginal value of exposing an additional viewer to an ad. To add these effects to this model would require a massive increase in complexity. Let it suffice to say that, in terms of the effects of increasing competition, this would tend to lower the price paid per viewer, but raise the price paid for each additional viewer contact at least once, relative to the solution shown here.

3. For example, most older comedies had 26 to 27 minutes of program time. Typically in syndication an additional 4 minutes of programming time is replaced by advertising time. More recently, first runs have more advertising time, so syndicators cut less for the reruns. Currently "Mr. Belvedere" first runs have about 5:52 (minutes:seconds) to 6:52 minutes of advertising time (the network will exercise flexibility depending upon current advertising inventory). In rerun syndication, the same shows will have 6:30 in syndicator advertising plus 1:32 of available local advertising time, for a total of 8:02, a 17% to 37% increase. (Source: K. Burns, Creative Services, 20th Century Fox TV Syndication.)

4. The monopolist need not select symmetric advertising. Since audiences are



fixed, it need only have the average of its outlets' minutes equal to  $a/4b$ . The asymmetric case adds no additional intuition or complications, so we suppress it here.

5. This is true for the single outlet or dual outlet monopoly, with a caveat that if the dual outlet monopolist selects to be asymmetric, one set of viewers may face fewer advertising minutes.

6. If asymmetric equilibria are considered, there are three equilibria, each satisfying.

$$\min(m_1^{**}, m_2^{**}) = a/4b; \text{ and } p^{**} = a/2.$$

The three equilibria are  $m_2^{**} > m_1^{**} = a/4b$ ;  $m_1^{**} > m_2^{**} = a/4b$ ; and  $m_2^{**} = m_1^{**} = a/4b$ , with the entire audience watching outlet 1, watching outlet 2, and split equally between 1 and 2.

7. Subject to the caveat in note 5.

8. The interior is defined by  $A^i > 0$  for all  $i$ . Desired "intermissions" may be created by some ads. This might lead to some convex sections; but even if it did, it would not change the qualitative conclusions.

9. It appears reasonable to assume that if a firm broadcasts a large enough (but finite) number of advertising minutes, its audience will fall to 0. This ensures the existence of a global minimum at (infinitely many) finite points of the domain. Further, for any firm  $i$ , if we set  $m_i = 0$  and  $m_j$  high enough that  $A^i = 0$  for all  $j \neq i$ ,  $A^i$  reaches a global maximum.

10. Fix  $m_i$  (for all  $j \neq i$ ) at some arbitrary level  $m_j^0$  in the zone of potential interior equilibria. Note that  $q^i$  is twice continuously differentiable since  $A^i$  is twice continuously differentiable. Given the strict concavity of  $A^i$  in the interior,  $A_{ii}^i < 0$ . This along with the restriction  $A_i^i < 0$  permits us to derive  $q_{ii}^i = 2A_i^i + A_{ii}^i m_i < 0$ . Now evaluate  $R_i^i$  at  $q_{ii}^i = 0$ . This may be seen to be  $-b \left[ \sum_{j \neq i} q_{ji}^i \right] q^i < 0$  since

$q^i > 0$  for  $j \neq i$ . Since  $R^i$  is concave and differentiable and  $R^i > 0$  for some  $m_i$ ,  $R^i = 0$  if  $m_i = 0$ ; and  $q^i$  is declining in  $m_i$ , by the mean value theorem the optimal  $m_i$  is such that  $q^i > 0$ .

11. It should also be clear that it is not driven by the assumption that the equilibrium is Nash in minutes: nonzero conjectural variations and other controls can also be accommodated.

12. See [4] for technical details.

13. It is likely that increased competition will reduce both  $q^i$  and  $q^j$ . As the number of viewing options increases, it is likely that the rate at which firm  $i$  loses audience by increasing its advertising minutes will rise. Similarly, the rate at which each other firm gains audience due to this action will fall as the diversion from firm  $i$  is distributed among more firms.

14. See note 10.

15. This is holding program "quality" and "diversity" effects constant.

16. In the Appendix, we generalize the model to allow for a positive audience subscription price. The same audience diversion effects are found to persist in this case. The technical conditions for the oligopoly price to actually rise with an increased number of firms become more cumbersome, but the underlying intuition is unchanged. This suggests that the results generalize to pay television and even to printed media if more pages of advertising are viewed negatively by a sufficient number of readers. It should be added, advertisers' and viewers' interests are not as directly opposed when there are subscription fees. This is because two factors — advertising minutes and subscription price — influence viewer preferences.

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