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A Note on Complementary Goods Mergers between Oligopolists with Market Power: Cournot Effects, Bundling and Antitrust

Abstract: Antitrust policy in the US and EU toward non-horizontal mergers between oligopolists is based on a strong presumption of Cournot effects and/or improvements in consumer welfare through post-merger bundling. We show that complementary goods mergers between firms that possess market power in their respective components markets do not always assure either. The analysis underscores the importance of fully specifying the nature of pre-merger rivalry among all market participants and the assumed distribution of consumer preferences when making predictions about the likely effects of such transactions.

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1 Introduction

Our point of departure is several policy statements issued by U.S. and EU authorities since the early 2000s about complementary goods mergers between firms which possess market power (earn high shares and high profit) in their respective

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product or component markets.[1] The presumptive benefits of such mergers through Cournot Effects ("CEs") and/or bundling are at best overreaching and at worst inaccurate because the statements stem from an oversimplification of the demand and/or market structure conditions which are necessary for CEs to exist.

1 These include:

"…when there are no economies of scope, when two producers of complementary products merge they may offer a lower price for a bundle of those products because the merger solves a "double-marginalization problem… This is the so-called "Cournot effect" … is all the more likely in those instances where the merging firms had been exercising a degree of market power before the merger. U.S. Antitrust Division submission for OECD Roundtable on Portfolio Effects in Conglomerate Mergers, Range Effects: The United States Perspective ("OECD Roundtable"), October 12, 2001, p. 11. http://www.justice.gov/atr/public/international/9550.htm.

To the extent the merging parties enjoyed large market shares and market power in complementary goods, there will be a tendency for prices to decline post merger … fears that a conglomerate merger involving portfolio effects would lead to a welfare reducing type of price discrimination involving tying or bundling could be a thin reed to lean on as the sole rationale for blocking the merger. Ibid, pp. 30–31.

A firm may bundle its product with a complement in order to soften competition. Bundling in this case increases the profits of all participants in the market… An easy way to detect whether softening competition is the motivation for bundling is to look at competitors' reactions to the bundle: If competitors are complaining about the possibility, we can be pretty sure that it is not serving to soften competition. Ibid, pp. 30–31.

To the extent a merger of complements gives the merged firm the incentive to lower prices because it causes the firm to internalize the negative externalities associated with higher prices (the so-called Cournot effect), it moves prices in the right direction – toward marginal costs – enhancing allocative efficiency through the elimination of double marginalization and benefitting consumers with lower prices and increased output." "We simply could not identify any conditions under which a conglomerate merger, unlike a horizontal or vertical merger, would likely give the merged firm the ability and incentive to raise price and restrict output. William Kolasky, [then Deputy Assistant Attorney General U.S. Department of Justice], Conglomerate Mergers and Range Effects: It's a Long Way from Chicago to Brussels, November 9, 2001.

Improved coordination between suppliers of *complementary* goods is an essential aspect of efficiency. Such improved coordination not only raises the parties' joint profits, but tends to increase overall efficiency as well through lower prices or improved quality. This externality between the parties could be better internalized by their vertical [stet] merger… OECD Policy Roundtables, Vertical Mergers, 2007, United States submission, pp. 239–248.

when producers of complementary goods are pricing independently, they will not take into account the positive effect of a drop in the price of their product on the sales of the other product. Depending on the market conditions, a merged firm may internalize this effect and may have a certain incentive to lower margins if this leads to higher overall profits (this incentive is often referred to as the "Cournot effect"). Official Journal of the European Union, October 18, 2008, paragraph 117.

CEs are the removal of a pre-merger pricing externality. In the context of two firms that each sell a product complementary to that sold by the other, neither internalizes the effect that its own price has on the demand for the other's product. This leads to a phenomenon called "double marginalization"; each firm applies a margin to its own product without accounting for the reduction in demand for the complementary product sold by the other firm. If the two firms merge, the combined entity will account for this pricing externality when it sets prices. Even though complementary goods mergers in oligopoly markets may simultaneously reduce competition from rivals, lower prices can still result if CEs are large enough.

Neither the U.S. antitrust agencies nor the EU have published the underlying demand and market structure conditions needed to generate CEs and/or lower prices, including situations where the merged firm bundles. As a result, merging parties may claim efficiencies from CEs without providing much analytical support,[2] and regulatory authorities pre-disposed toward these claims are likely to discount competitors' complaints.[3]

We consider three cases that assume, as did Cournot (1838), fixed-proportions demand for two components. In these cases, a non-horizontal merger between two oligopolists with market power produces no CEs. In all three, the underlying model of pre-merger oligopoly is two price-setting firms; each sells one of the components and competes against multiple firms that have no individual market power and price at marginal cost. The three cases differ with respect to the distributions of consumers' preferences for the individual components. In one case, the merger has no effect on consumer welfare, while in the other two it falls *and* rival sellers are made worse off. The latter two demonstrate that competitor complaints about complementary goods mergers between oligopolists do not always signal efficiencies, including CEs. These examples also illustrate that prior to positing a merger's effect on competition, the underlying model of oligopoly behavior and distribution of consumers' preferences should be specified.

Section 2 provides a brief review of the relevant literature and Section 3 presents the three cases.

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2 In criticizing the EU's decision to challenge the GE-Honeywell merger Hal Varian concludes that "GE-Honeywell ran afoul of 19th-century thinking." Specifically:

> [A]ntitrust authorities rightly frown on companies' coming together to set prices, since the effect is often anticompetitive. On the other hand, if the products are highly complementary and are produced in highly concentrated industries, producers left to their own devices may set prices too high because of the "Cournot effect." [New York Times, June 28, 2001].

3 Supra note 1.

2 Relevant literature

Existing literature on complementary goods mergers between oligopolists with market power predicts CEs when the merging firms' inputs are combined or consumed in fixed proportions. Relevant papers include Economides and Salop (1992) (E&S), Choi (2008), Alvisi et al. (2011) (AC&P). In some of these (E&S, AC&P), post-merger prices may increase either because CEs do not exist or they are not large enough to offset the effect of reductions in the number of rival sellers. For example, the relevant part of AC&P for our paper is based on separate versus joint ownership of high-quality hardware with high-quality software and of low-quality hardware with low-quality software, a market structure which they label "full leadership." In this case, and for their demand specification, they find that CEs do not exist because each (high or low) quality version of a component (hardware or software) is a gross substitute for the same-quality version of the other component.[4] Consequently, their model predicts that under full leadership, divesting the two integrated systems sellers into four firms that produce individual components can improve competition and lower prices. Their results apply even when the firm that produces the high-quality system is divested into two separate component producers and the other integrated firm is left intact.[5]

CEs are also an integral part of the literature that addresses firms' decisions to make components compatible or incompatible. Denicolo (2000) models two firms each of which sells two complementary components; he breaks one of the firms into two independent sellers of individual components. He varies the degree of product differentiation for one of the components and analyzes whether the remaining integrated firm will choose to make its version compatible or incompatible with the complementary component sold by one of the new unintegrated producers. The firm's decision hinges in part on the amount of double marginalization that is removed. For example, when the versions of this component are undifferentiated, the profit-reducing effect of double marginalization dominates, leading the integrated producer to choose incompatibility.

4 The absence of CEs under our demand specifications results from pre-merger price competition between a high-quality version of a component and homogenous low-quality versions sold by multiple firms that engage in pure Bertrand pricing.

5 AC&P also consider the case in which each integrated firm sells one high- and one low-quality component; in this case divestiture may lead to double marginalization. Under this setup, their model predicts that double marginalization (i.e., "tragedy of the anticommons" in their terminology) will more than offset the benefits from the increase in competition.

Dari-Mattiacci and Parisi (2006) ("D-M&P") demonstrate that with N components needed to form a system a sufficient condition to remove CEs from a complementary goods merger is pure Bertrand competition among symmetric competitors in N–1 of the N component markets. Their result rests on the assumption of homogenous preferences in all components markets meaning there is neither vertical nor horizontal differentiation.[6] Given pure Bertrand competition, firms exercise no market power and price remains equal to marginal cost even after a merger of two complementary components producers. D-M&P do not address whether CEs always occur when market power is exercised pre-merger in all markets.

3 Three examples of non-horizontal mergers that produce no CEs even though the merging firms exercise significant market power

To set the stage, consumers are assumed to purchase one unit each of two complementary components combined to form a system. Components are fully compatible and are either high or low quality. Pre-merger, each high-quality component is sold by a separate firm while two or more firms sell each of the low-quality versions. Consumers can purchase a low-quality component at a price indexed to zero or they can purchase a high-quality component at a positive price. Each consumer's valuation of a component is the premium he/she would pay for high quality over low quality up to a maximum normalized to unity. Consumer j will purchase the high-quality version of component i if and only if:

$$v_{ij} - p_i > 0 \tag{1}$$

where v_{ij} is the premium that consumer j would be willing to pay for the high-quality version.

All component producers have zero marginal cost and are assumed to engage in (quality-differentiated) Bertrand price competition. Because two or more low-quality producers compete in the sale of each component, each sets a price equal to its marginal cost of zero.

We consider three different uniform distributions of consumer preferences for the two high-quality components: (1) perfectly positively correlated, (2)

6 Vertical differentiation refers to a situation where all consumers are willing to pay a premium for a particular version of a component. Horizontal differentiation means *some* consumers would pay a premium for one version of a component while others would pay a premium for a different version.

perfectly negatively correlated, and (3) imperfectly negatively correlated.[7] For each correlation, we derive pre-merger and post-merger prices by combining the two high-quality producers into a single firm. We also report pre- and post-merger values for consumer surplus and producer surplus.[8]

3.1 Perfectly positively correlated preferences

With perfectly positively correlated preferences, consumers' valuations are distributed uniformly on the 45° line spanning (0,0) to (1,1). Consumers evaluate the four different systems which are defined by combinations of high- and low-quality components and choose that system which offers the highest net utility (net of cost). As noted, when forming a system each consumer decides whether or not to purchase a high-quality component based solely on its value and price compared to the low-quality version. Since valuations for each high-quality component are distributed uniformly in the interval [0,1], the implied demand for high-quality component i ($i=1, 2$) is:

$$q_i(p_i) = 1 - p_i \qquad [2]$$

7 We skip the case of zero correlation, usually modeled as preferences for the two high-quality components distributed uniformly on a unit square. This preference distribution also leads to no CEs after a merger of the two high-quality component producers, and like case B below generates a loss of consumer surplus. Pre-merger, optimal high-quality component prices are ½, as a result, one-quarter of the population purchases each of the four system types. While a merger between the two high-quality producers followed by pure bundling does not change individual component prices, post-merger one-half of the population purchases the high-quality system while the other half purchases a system comprised of only low-quality components. The merged firm captures one-half of the consumer surplus that was earned pre-merger by the consumers that purchased a hybrid system. Both Einhorn (1992) and Matutes and Regibeau (1988) model the zero-correlation case; however, their models include at most four independent producers, each with some market power.

8 Although we report pre- and post-merger producer surplus, both the U.S. Horizontal Merger Guidelines (2010) and EU Non-Horizontal Merger Guidelines (2008) endorse a consumer welfare standard for evaluating transactions. ("Mergers should not be permitted to create, enhance, or entrench market power or to facilitate its exercise. A merger enhances market power if it is likely to encourage one or more firms to raise price, reduce output, diminish innovation, or otherwise harm customers as a result of diminished competitive constraints or incentives." – DOJ, FTC Horizontal Merger Guidelines, August 2010, p. 2; "Effective competition brings benefits to consumers, such as low prices, high quality products, a wide selection of goods and services, and innovation. Through its control of mergers, the Commission prevents mergers that would be likely to deprive customers of these benefits by significantly increasing the market power of firms." – Official Journal of the European Union, 2008/C 265/07, October 2008, paragraph 10.)

which is independent of the price of the other high-quality component. Firm i then solves the profit maximization problem:

$$\max_{p_i}(1 - p_i)p_i \qquad [3]$$

Solving eq. [3] leads to an optimal component price $p^* = ½$ for each component. The price of a system with the two high-quality components is hence $2p^* = 1$, or $p^* = ½$. Half of all consumers purchase either the high-quality or low-quality system. Consumer surplus is 1/4 and profits equal ½ (1/4 for each high-quality producer).

A merger between the producers of the high-quality components does not alter prices or the distribution of consumers by type of system. The merged firm solves the following profit-maximization problem:

$$\max_{P}(1 - P/2)P \qquad [4]$$

which results in a system price of 1. Half of all consumers continue to purchase the high-quality system while the other half purchase the low-quality system. The merger is neither profit-enhancing nor does it change consumer welfare.[9] Three conditions, (i) all components are compatible, (ii) vertical differentiation exists between the high- and low-quality versions of a component and (iii) pure Bertrand competition among sellers of a low-quality version, are jointly sufficient for the absence of CEs. Case A satisfies all three conditions.[10]

3.2 Perfectly negatively correlated preferences

With perfectly negatively correlated preferences, all consumers place a value of unity on a high-quality system. Those placing a value of one on one high-quality component value the other high-quality component at zero, those valuing one at 3/4 value the other at 1/4, etc. This means that all consumers are located on the diagonal from (0,1) to (1,0) and implies that demand for the high-quality system is perfectly inelastic at $Q = 1$ for system prices less than or equal to 1. Pre-merger,

9 Presumably such a merger would be motivated by objectives outside of those addressed by this note.

10 Without suppliers of low-quality components, pre-merger each component monopolist accounts for the other's price when setting its own. Profits for each monopoly producer would equal $(1 - P/2)p_i$, for $i = 1, 2$, leading to identical reaction functions $p_i = 1 - p_j/2$, optimal pre-merger prices for each component of 2/3, and a system price equal to 4/3. Their combination results in CEs because the merged firm maximizes total system profit of $(1 - P/2)P$ leading to an optimal system price of 1. This same result is obtained when low-quality component producers compete pre-merger but high- and low-quality components are incompatible.

each high-quality component producer faces the implied demand function expressed by eq. (3), leading to the optimal price of ½. Unlike case A, however, no consumers purchase systems comprised of just high- or low-quality components. Instead, only hybrid high-low quality systems are bought at a system price of ½. The high-quality component producers each earn positive profit and a market share of 0.5. Profits for each are 1/4, total firm profits are ½, consumer welfare is 1/4, and total surplus equals 3/4.

Post-merger, the integrated firm maximizes profit by bundling the two high-quality components and charges a system price equal to 1. While the implicit prices for the two high-quality components remain at ½, all consumers pay \$0.50 more for a system. Although total surplus increases from 0.75 to 1, this gain occurs entirely at the expense of consumers who pay more for a less-preferred system. As with the first case A, market power and positive mark-ups for the two high-quality producers generate no CEs. Further, the producers of low-quality components lose all their market share suggesting their concerns should be credited were they to complain about the merger.

3.3 Imperfectly negatively correlated preferences

In this third case, we assume that consumer's preferences are distributed on a unit circle, the northeast quadrant of which spans the same coordinates which define the endpoints of the diagonal in case B. Relaxing the assumption of perfectly negatively correlated preferences ensures that all consumers value a system comprised of the two high-quality components by at least \$1.00, with a maximum valuation of \$1.42.[11] The Appendix demonstrates that the optimal pre-merger prices for the two high-quality components are 0.65. At these prices, approximately 10% of consumers purchase the high-quality system, and the remainder purchase a hybrid high-low quality system. The average system price paid by all consumers equals 0.7144. Each high-quality producer achieves a 55% share and earns profit equal to 0.36 (see Appendix).

Post-merger, the combined firm maximizes profit by mixed bundling,[12] setting prices for the high-quality system and the individual components at

11 This amount is the sum of the valuations for that consumer who values the two high-quality components equally (and maximally across all consumers) and is derived from the formula for the circle, $x^2 + y^2 = r^2$ where $x = y$ and $r = 1$, i.e. $2x^2 = 1$. Solving for x results in $x = 0.71$ and $2x = 1.42$.

12 The Appendix also shows the merged firm would not choose to pure bundle. While its profits from pure bundling exceed the sum of the two firms' pre-merger profits and consumer welfare increases, mixed bundling generates even greater profits because it allows the firm to

1.2356 and 0.9618 respectively.[13] While the types of systems purchased remain the same, the composition of customers by system type changes. High-quality systems account for 65% of purchases and hybrid systems account for 35%, making the average post-merger system price equal to 1.14. Further, because mixed bundling shrinks from 45 to 17.5% the combined share of producers selling a low-quality component, complaints about the merger by competitors are accurate predictors of the merger's likely harm to competition.

The overall increase in average system price does not reveal the underlying welfare effects that occur in the different customer segments. The 35% of consumers who purchase a hybrid system before and after the merger pay 47% more, while the near-10% who purchased a high-quality system enjoy an approximate 5% price decline. This leaves 55% of consumers who switch from a hybrid system to a high-quality one. A simple comparison of the system prices they pay post- vs. pre-merger fails to measure the change in their welfare because it does not account for any additional value they place on ownership of a high-quality system.

The effective post-merger price increase to them can be measured by the increase in the implicit price of the high-quality component which they purchased pre-merger. For example, a consumer who values the two high-quality components at 0.8 and 0.5 would purchase only the first one pre-merger and earn consumer surplus of 0.1478 (0.8 – 0.6522). Post-merger, this consumer pays 1.2356 for the bundle. Since the valuation he/she places on the other high-quality component is only 0.5, the implicit post-merger price of that component purchased pre-merger is 0.7356 ($= 1.2356 - 0.5$). This represents a 12.78% increase in the implicit price of the high-quality component. Expressed in terms of this component, this individual's consumer surplus falls from 0.1478 ($= 0.8 - 0.6522$) to 0.0834 ($= 0.8 - 0.7356$), a decrease of 43.6%.

Not all consumers who switch to the high-quality system incur an implicit price increase. Indeed, those with relatively "high" valuations for both components enjoy an implicit price decline. Post-merger, they earn more consumer surplus from the purchase of the bundle than they obtained pre-merger from the

price-discriminate and charge a higher price to those consumers who place a large value on only one of the high-quality components.

13 Even though the price of the bundle under either pure or mixed bundling is less than the sum of the pre-merger component prices, this result does not reflect the presence of CEs because no pricing externalities are internalized by the merger. A necessary and sufficient condition for the presence of CEs if all components are compatible is lower post-merger prices of the two high-quality components sold only separately. In case C, separate components pricing post-merger results in prices for the two high-quality components which equal their pre-merger prices. See Appendix, footnote 16.

purchase of one high-quality component. For example, a consumer who values the two high-quality components at 0.80 and 0.60 purchases only the former pre-merger and earns consumer surplus equal to 0.1478 (= 0.80 − 0.6522). But, post-merger he/she purchases the high-quality system and earns surplus of 0.1644 (= 1.40 − 1.2356), a decrease of 11.2%. The implicit price decline for the component purchased pre-merger is 0.0166 (= 0.1644 − 0.1478) or 2.5%. The Appendix shows that the mass of consumers made better off from switching are those with component valuations equal to 1.3956 or higher.[14] Table 1 displays consumer mass and the dollar surplus gain/loss for each customer segment and across all consumers. In the Table, the column "CS increase" shows that about 20% of consumers gain, while the two columns "CS decrease" show that approximately 80% lose. The dollar gains to those consumers who experience increases in surplus total 0.011 while dollar losses to those suffering a decline equal 0.176, making the total consumer welfare loss 0.165 (See Appendix).

Table 1: Consumer choice and the effect on consumer surplus, proportions of consumers[a]

		Post merger system			
		Hybrid[b]	High-quality		
		CS decrease	CS decrease	CS increase	Total
Pre-merger system	Hybrid[b]	35.3%	44.0%	11.2%	90.4%
		−0.109	−0.067	0.005	−0.171
	High-quality	n.a.	n.a.	9.6%	9.6%
				0.007	0.007
	Total	35.3%	44.0%	20.8%	100.0%
		−0.109	−0.067	0.012	−0.164

Notes: [a]Totals may not add due to rounding. [b]Purchase of one high-quality and one low-quality component.

4 Conclusion

In the introduction, we reference statements or guidelines issued by antitrust officials in the United States and Europe that strongly suggest mergers among complementary goods oligopolists with market power will typically produce lower prices and consumer benefits in the form of CEs and/or bundling. To show that these effects are not ubiquitous we posit a complementary goods

14 That is, consumers with component valuations of at least (0.5834, 0.8122) or (0.8122, 0.5834).

merger that occurs within an oligopoly market structure that we believe is plausible and/or occurs frequently.

In that market structure the merging firms meet thresholds for the two indicia that are most commonly used to assess the presence of market power – large shares and positive profits. Yet, no CEs occur when (i) all components are fully compatible; (ii) the dominant producers selling each high-quality version are vertically differentiated from those selling low-quality versions; and (iii) the sellers of the low-quality versions engage in pure Bertrand competition.

We also show that within this market structure the often-presumed benefits of post-merger bundling do not always transpire. Depending upon the distribution of consumers' preferences, the merged firm may be indifferent between pure components pricing and bundling (case A), prefer pure bundling (case B), or choose mixed bundling (case C). Cases B and C reflect situations where the merged firm selects a form of bundling that both reduces consumer welfare and excludes competitors. In these situations, rather than signaling efficiencies to be realized by the merged entity, rivals' complaints are an accurate harbinger of the transaction's likely effects on consumer welfare.

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Appendix

Consider one high-quality producer for each of the two components (1 and 2), denoted as 1H and 2H, that comprise a system. Also, assume the presence of at least two undifferentiated low-quality producers of these same components, denoted as 1L and 2L, respectively. Mixed systems are denoted by HL for 1H,2L and LH for 1L,2H. The "high quality system" is labeled HH for 1H,2H.

Let Ω denote the positive quadrant of the unit circle. Since the circumference of the circle equals 2π, the length of Ω is $\pi/2$. Since the interval length is $\pi/2$, the consumer density is $2/\pi$. Given this, the length of an any arc contained in Ω (corresponding to a particular set of consumers' preferences), multiplied by $2/\pi$, equals the proportion of the population with those preferences.

Equilibrium when components are priced independently

Consider the profit maximization problem of firms 1H and 2H. Each consumer has valuations of the high-quality components denoted v_{1H} and v_{2H} respectively (we suppress the subscript j for the consumer's identity). A consumer purchases a system that contains component 1H (or 2H) when $v_{1H} > p_{1H}$ (or $v_{2H} > p_{2H}$). Figure 1 illustrates the equilibrium purchase decisions of consumers.

An angle on a circle measures *one radian* if the arc length is equal to the radius of the circle, $r = 1$. So d radians can be written as $d = L/r = L$, where L is the arc length. Also on a unit circle, the sine of an angle is equal to the length L_y of the y-component ("rise"), and its cosine is equal to the length L_x of the x-component ("run"): $L_y = \sin(d)$ and $L_x = \cos(d)$. (In our context, L_y and L_x correspond to the vertical "v_{1H}, p_{1H}" coordinate and the horizontal "v_{2H}, p_{2H}" coordinate, respectively.)

An angle on the unit circle can be written as $d = \cos^{-1}(L_y)$ or $d = \sin^{-1}(L_x)$. Since $L = d$, $L = \cos^{-1}(L_y) = \sin^{-1}(L_x)$. The length of the arc is the arccos of the horizontal "v_{2H}, p_{2H}" coordinate or the arcsin of the vertical "v_{1H}, p_{1H}" coordinate. The total proportion of consumers to which each firm sells is then $\text{Demand(Firm 1H)} = 1 - (2/\pi)\sin^{-1}(p_{1H})$ and $\text{Demand(Firm 2H)} = (2/\pi)\cos^{-1}(p_{2H})$. Moreover, since for any numeric value v, $\sin^{-1}(v) + \cos^{-1}(v) = \pi/2$, it follows that $\text{Demand(Firm 1H)} = (2/\pi)\cos^{-1}(p_{1H})$. Therefore, the profit maximization problem for each firm is to set a price p_{iH} to solve:

$$\Pi_{iH} = p_{iH} \times \left[\frac{2}{\pi}\cos^{-1}(p_{iH})\right], \ i = 1, 2 \qquad [5]$$

Differentiating with respect to p_{iH} and using $\frac{\partial \cos^{-1}(v)}{\partial v} = -\frac{1}{\sqrt{1-v_i}}$ yields:

$$\frac{2}{\pi}\cos^{-1}(p_{iH}) - \frac{2}{\pi} \times \frac{p_{iH}}{\sqrt{1 - p_{iH}^2}} = 0, \ i = 1, 2 \qquad [6]$$

Solving eq. [6] leads to prices $p_{1H} = p_{2H} = 0.6522$, firm profits of 0.3572 and a combined profit of 0.7144. Only 9.56% of the consumers purchase the

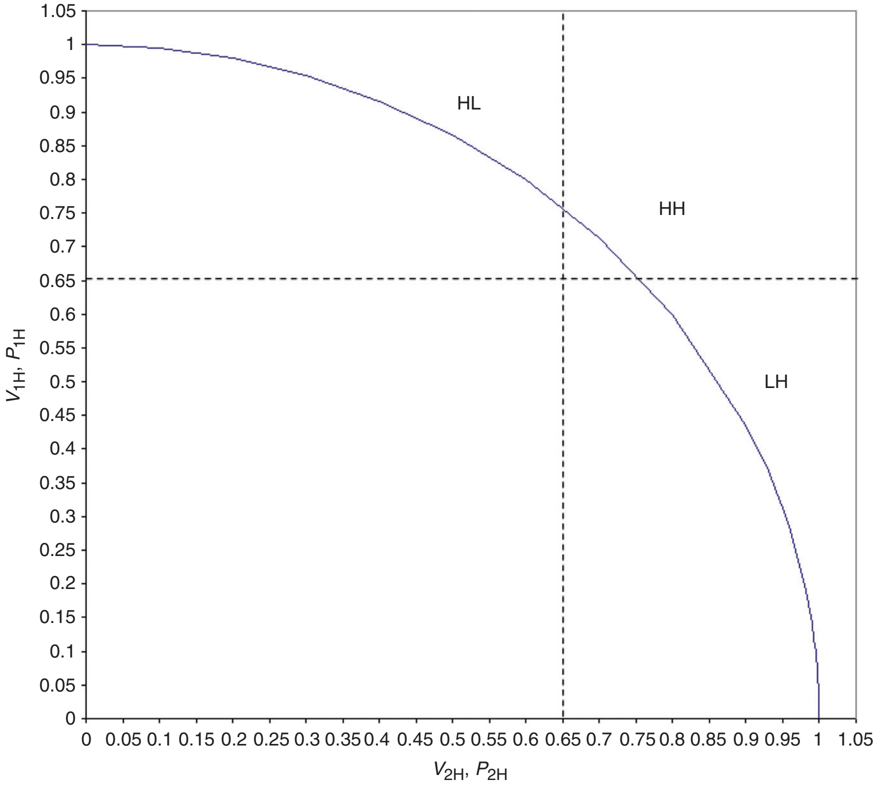

Figure 1: Equilibrium with independent pricing of high-quality components

high-quality system while 45.21% purchase a mixed system. Each high-quality component is thus sold to 54.77% (=9.56 + 45.21) of the population. Total consumer surplus under this equilibrium equals 0.25 (0.24 for consumers purchasing mixed systems plus 0.01 for those buying systems with both high-quality components).[15] Total surplus across the two component markets equals 0.9652 (=0.7144 + 0.2508).

15 At component price $p^* = 0.6522$, total demand for either high-quality component is $q^* = (2/\pi)\cos^{-1}(p^*) = 0.5477$. Inverse demand is $\text{Demand}^{-1}(q) = \cos\left(q\frac{\pi}{2}\right)$. For any one component total consumer surplus (CS) is $\int_0^{q^*} \text{Demand}^{-1}(q)dq - p^*q^* = 0.1254$. Twice this amount, 0.2508, is total CS. On Ω, consumers who value the high-quality component more than $v^* = \sqrt{1-p^{*2}} = 0.7583$ do not purchase the other high-quality component. Their mass is $q_0 = \text{Demand}(v^*) = 0.4521$. Consumers who purchase both high-quality components measure $q^1 = q^* - q^0 = 0.0956$ and earn CS equal to $\int_0^{q^*} \text{Demand}^{-1}(q)dq - p^*q^1 = 0.0052$. By symmetry, their CS on both components is 0.0104. CS earned by consumers who purchase only one high-quality component is $0.2508 - 0.0104 = 0.2404$. See also Table 2.

Post-merger pure bundling

After the two high-quality producers merge, and in the absence of bundling, their optimal individual component prices remain the same and no CEs occur.[16] If the merged firm chooses to pure bundle, consumers must purchase either a high- or low-quality system. If the pure bundle price of the high-quality system equals P, consumers who purchase it are those located on Ω to the northeast of the line that slopes downward from (0, P) to (P, 0). Figure 2 illustrates purchase decisions of consumers given an arbitrary price of 1.2 for the pure bundle HH.

With pure bundling, system demand is given by[17]:

$$Q(P) = \begin{cases} 1 & \text{if } P \le 1 \\ 1 - 2\left[\frac{2}{\pi}\cos^{-1}\left(\frac{P+\sqrt{2-P^2}}{2}\right)\right] & \text{if } P > 1 \end{cases} \qquad [7]$$

Based on this demand curve profits can be expressed as a function of the bundle price P. The horizontal axis starts at the price of 1 because all consumers value the high-quality system by at least this amount. Also, at a price of $\sqrt{2}$, the bundle price line is tangent to the unit circle meaning that profits are zero. This

16 Recall the demand for high-quality component j is given by $(2/\pi)\cos^{-1}(p_{jH})$ for $j = 1,2$. Thus the demand for 2H is independent of the price of 1H and vice versa. Given independent demands and absent bundling, the post-merger first-order conditions for profit maximization are identical to the pre-merger first-order conditions, implying identical pre- and post-merger prices.

17 Let $Q(P)$ denote the mass of consumers who purchase the bundle at price P. For these consumers $v_{1H} + v_{2H} > P$. Let $F(P)$ be the mass of consumers for whom $v_{1H} + v_{2H} < P$. Thus $Q(P) + F(P) = 1$, and bundle demand is $Q(P) = 1 - F(P)$. The bundle price is represented as a line with a slope of –1 along which $x + y = P$. (The x and y respectively correspond to v_{2H} and v_{1H}.) The unit circle is defined by the equality $x^2 + y^2 = 1$. For any value of P strictly between 1 and $\sqrt{2}$, the bundle price line and the unit circle intersect (and the bundle price line bisects the unit circle) at two distinct points (x^*, y^*) and (x^{**}, y^{**}). Let $y^{**} > y^*$. Half of the consumers for whom $v_{1H} + v_{2H} < P$ are located on the lowermost part of Ω below the lower intersection point (x^*, y^*) and the other half are located to the left of (x^{**}, y^{**}). Then, (x^*, y^*) can be derived as follows: The price line is $y = P - x$ and the unit circle is $y = \sqrt{1 - x^2}$. Equating these, $P - x = \sqrt{1 - x^2}$ leads to the quadratic expression $2x^2 - 2Px - (1 - P^2) = 0$, which has a root at $x^* = \left(P + \sqrt{2 - P^2}\right)/2$ and $y^* = 1 - x^*$. By the trigonometric discussion immediately preceding equation [5], the length of the arc from the "base" of Ω to the point of intersection (x^*, y^*) equals $\cos^{-1}(x^*)$. Half of the mass of consumers who do not buy the bundle equals this arc length normalized by $2/\pi$. This mass is doubled to account for an equal mass of consumers at the "apex" of Ω who do not buy the bundle. Thus $F(P) = 2(2/\pi)\cos^{-1}\left[\left(P + \sqrt{2 - P^2}\right)/2\right]$. Since $Q(P) = 1 - F(P)$, the demand for the bundle at price P equals $1 - 2(2/\pi)\cos^{-1}\left[\left(P + \sqrt{2 - P^2}\right)/2\right]$.

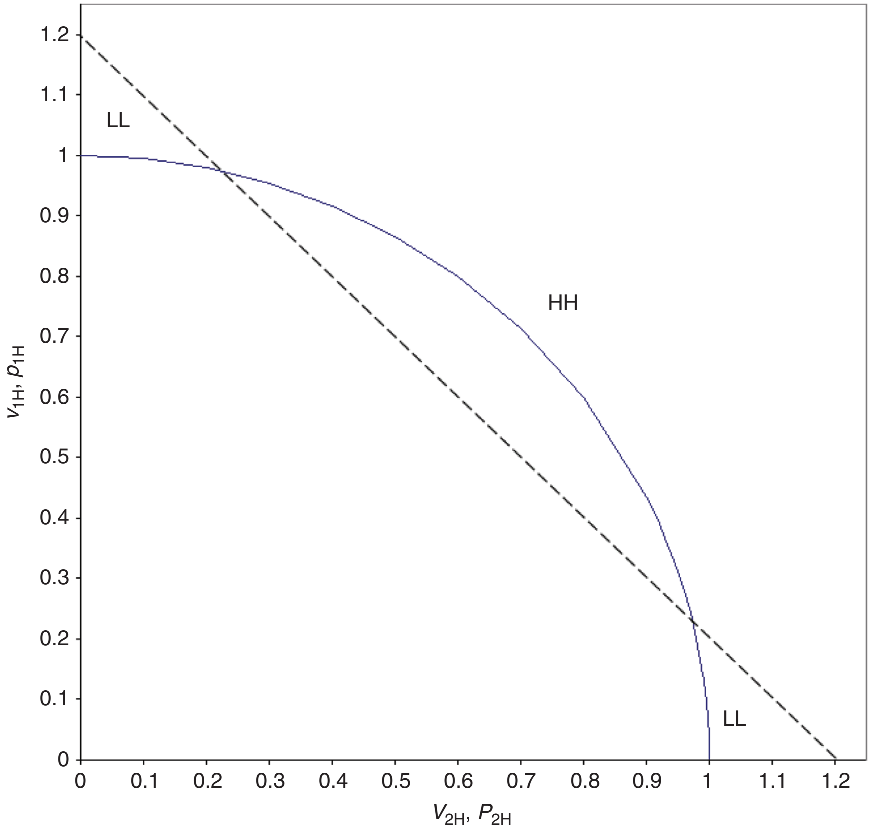

Figure 2: Pure bundling of components

profit function is maximized at bundle price $P^* = 1$ corresponding to an equilibrium quantity $Q^* = Q(P^*) = 1$.[18]

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18 The formal argument as to why the post-merger pure bundling equilibrium occurs at $(P^*, Q^*) = (1,1)$ is as follows. (There is a kink in the demand curve at $Q = 1$, so the following calculus defines "partial derivative with respect to P at $P = 1$" as P approaches 1 from above.) Elasticity of the system demand $Q(P)$ is given by $\varepsilon = Q'(P)P/Q(P)$. At $P = 1$, $Q = 1$, this elasticity simplifies to $\varepsilon = Q'(P) = \text{num}(P)/\text{den}(P)$ where $\text{num}(P) \equiv 2\sqrt{2}P\left(1 - \frac{P}{\sqrt{2-P^2}}\right)$ and $\text{den}(P) \equiv \left(\sqrt{1 - P\sqrt{2-P^2}}\right)\pi$. Moreover, ε^2 simplifies to $\frac{16}{(2-P^2)\pi^2}$. Therefore at $P = 1$, $\varepsilon^2 = 16/\pi^2$ which implies $|\varepsilon| = 4/\pi > 1$, i.e. the demand is elastic at $P = 1$. We will first argue that the merged firm does not have an incentive to raise price above $P = 1$. At $P = 1$, elasticity exceeds 1 and marginal cost is constant at zero. Therefore, if the merged firm priced at $P > 1$ it would lose quantity (and revenue) without avoiding cost. Since $Q(P)$ is concave, demand elasticity

Profits at the equilibrium pure bundle price are equal to 1, which are higher than the merged firm's profits of 0.7144 under unbundled pricing. Consumer surplus is calculated as $CS = \int_0^{Q^*} P(Q)dQ - \Pi^*$, where $\Pi^* = \Pi(P^*) = 1$ and $P(Q)$ is the inverse demand function. This calculation yields CS $=$ 0.2732. However, although both consumer and producer surplus increases with the merger, pure bundling is not equilibrium behavior, as the next section clarifies.

Post-merger: mixed bundling

Mixed bundling is more profitable than pure bundling. Figure 2 shows that with a pure bundle price in excess of 1 (in the figure the pure bundle price depicted is 1.2) two "extreme" groups of consumers do not purchase the bundle. They are those who place a very low value on one high-quality component and a high value on the other. With pure bundling the firm charges a "low" bundle price equal to one to attract all consumers. However, because a significant number of consumers value the two high-quality components by an amount that significantly exceeds one, the potential exists for the merged firm to mixed bundle by setting a bundle price higher than one provided it can set individual component prices "close" to one and capture those consumers who place a high value on only one component.

For mixed bundling, we augment the notation by denoting the high-quality bundle price as P_{HH}, and p_{1H} and p_{2H} as the prices for each of the high-quality components when sold separately. A consumer will purchase the high-quality bundle provided (i) its value exceeds its price ($P_{HH} < v_{1H} + v_{2H}$) and (ii) the net value of buying only one high-quality component is less than the net value of buying the high-quality bundle ($v_{iH} - p_{iH} < v_{1H} + v_{2H} - P_{HH}$ for $i = 1,\ 2$). This leads to the inequalities: $P_{HH} - p_{1H} < v_{2H}$ and $P_{HH} - p_{2H} < v_{1H}$. If these inequalities do not hold, either of the mixed systems HL or LH will be purchased provided $p_{1H} < v_{1H}$ or $p_{2H} < v_{2H}$, respectively.

Lemma: *Under mixed bundling for a given bundle price, the merged firm will choose component prices so that consumers not purchasing the bundle purchase one of the two high-quality components.*

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increases with P meaning any $P > 1$ is dominated by $P = 1$. Next, we argue that the merged firm does not have an incentive to price below $P = 1$. Since all consumers have a reservation price of at least one, any $P < 1$ is dominated by $P = 1$. Stated technically, if elasticity > 1, and MC $=$ 0, there is an incentive to lower price and raise quantity because $MR > MC$. At the kink, however, lowering price does not lead to higher quantity, causing the same quantity to be sold for less. Thus, the merged firm will maximize profit at $P^* = 1$.

Proof: Intuitively obvious, formal proof available upon request.

The **Lemma** implies that given a bundled system price the stand-alone price for each component is determined by the intersection of the bundle price line and Ω. Therefore, the firm's profit function can be expressed as a function of a single parameter, the bundle price. Given the lemma above, profits under mixed bundling can be expressed as:

$$\Pi = P_{HH}\left(1 - 2 \times \tfrac{2}{\Pi}\cos^{-1}(p)\right) + 2p\left(\tfrac{2}{\pi}\cos^{-1}(p)\right)$$
$$\text{where } p = p_{1H} = p_{2H} = \tfrac{P_{HH}+\sqrt{2-P_{HH}^2}}{2}. \qquad [8]$$

P_{HH} denotes the bundle price and p_{1H} and p_{2H} are determined by the two intersections of the bundle price line and Ω.[19] The first term in eq. [8] is profits from sales of the bundle. The second term is profits from the sales of the individual components 1H and 2H. Solving the first-order conditions gives a bundle price of $P_{HH} = 1.2356$. From eq. [8], the individual component prices that maximize mixed-bundling profits are $p_{1H} = p_{2H} = 0.9618$. (Proof of the mixed-bundle equilibrium is available upon request.)

Profits under mixed bundling are 1.1389, 13.89% higher than profits with pure bundling (and 59.4% higher than pre-merger profits). Hence, mixed bundling strictly dominates pure bundling as a post-merger pricing strategy. Consumer welfare is 0.0857, significantly lower than the pre-merger consumer welfare of 0.2508.

Welfare comparison

Since the merged firm will choose to mixed bundle, the welfare effects of the merger are derived assuming this strategy.

The 90.46% of consumers who purchased mixed systems pre-merger drops by about 60% to 35.30% post-merger. This means that 55.16% (90.46–35.30 = 64.70–9.54) of consumers switch from the purchase of a mixed system to the bundle.

19 The profit function [8] follows from the bundle demand expression [7] and the individual expressions for the high-quality components [5]. The first term captures profits from bundle sales, and equals the product of the bundle price P_{HH} with the bundle demand expression [7], given that $p = \left(P + \sqrt{2 - P^2}\right)/2$ and $P = P_{HH}$. The second term captures profits from individual component sales and is identical to the profit expression [5], with p substituted for p_{iH}. The expression $p = p_{iH} = \left(P_{HH} + \sqrt{2 - P_{HH}^2}\right)/2$ is identically derived as, and identical to, the expression for x^* in footnote 17 above, with P_{HH} replacing P.

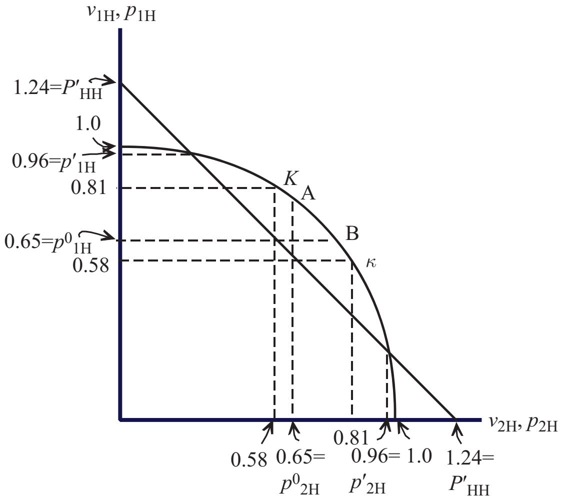

Figure 3: Equilibrium with mixed bundling

In Figure 3 consumers above 0.96 and to the right of 0.96 purchase mixed systems post-merger at a price equal to 0.96. This represents a 47.5% increase in the price they pay for a mixed system, and their consumer surplus falls from 0.1183 to 0.0090.

Consumers between A and B purchase the high-quality system both before and after. Their costs fall 5.27% from 1.3044 to 1.2356 and their consumer surplus increases from 0.0104 to 0.0169.

To calculate welfare effects for the approximately 55% switching from a hybrid system to the bundle, one must calculate their implicit price change expressed in terms of the price of the high-quality component which they bought pre-merger.

Let p^0_{iH} be the pre-merger price of high-quality component i purchased by a consumer who switches to the bundle post-merger, and as noted let P'_{HH} equal the price of the bundled system. Let the component valuations for this consumer be v_{1H} and v_{2H}. If the consumer purchased component 1H pre-merger, the implicit post-merger price paid for that component under mixed bundling equals $P'_{HH} - v_2$, which is the total price paid net of the value the consumer places on having good 2H as part of the system. The change in consumer surplus for this individual is thus $\Delta U = v_2 - P'_{HH} + p^0_{1H}$.[20]

20 Pre-merger utility is $v_1 - p^0_{1H}$; post-merger it equals $v_1 - (p'_{1H,2H} - v_2)$ when written to illustrate the change in the implicit price of 1H. Hence, $\Delta U = v_2 - P'_{HH} + p^0_{1H}$.

In Figure 3 consumers switching from a mixed system to a high-quality system are located to the left of A and above the bundle price line and to the right of B and above the bundle price line. Some of these consumers gain from the merger while some lose. Those who gain are located on the arc between K and A and the arc between B and κ. Consumers made worse off are those to the left of K and above the bundle price line, and symmetrically to the right of κ and above the bundle price line. The point K (symmetrically κ) defines $\Delta U = 0$. The exact coordinates for K are (0.5834, 0.8122) indicating a valuation for the bundle of 1.3956. At this point, pre-merger surplus (0.8122 minus 0.6522) equals post-merger surplus (1.3956 minus 1.2356). Geometrically, at point K, $P'_{HH} - p_{1H}$ is the distance 1.2356–0.6522 on the vertical axis, which is equal to $v_2 = 0.5834$ (=1.2356 – 0.6522) on the horizontal axis so $\Delta U = 0$. Point κ is analogously constructed for the value of good 1.

Change in consumer surplus by consumer type

Table 1 in the text presents the four categories of consumers by system type, pre- versus post-merger and displays both their mass and change in their consumer surplus by type. Table 2 below is an expanded version of Table 1; it includes a formulaic calculation of consumer surplus pre- versus post-merger by category. For high-quality components, let $q = \theta(p) = (2/\pi)\cos^{-1}(p)$ be individual component demand, $Q = \Theta(P)$ be the bundle demand defined in eq. [7]; let $p = \psi(q) = \cos(q\pi/2)$ for $0 < q < 1$ be the inverse demand for a high-quality component and $P = \Psi(Q) = \cos((1+Q)\pi/4) + \sin((1+Q)\pi/4)$ for $0 < Q < 1$ be the inverse demand for the high-quality bundle.[21] Additionally, let:

$p^* = 0.6522$; profit-maximizing pre-merger component price
$p = 0.9618$; post-merger stand-alone component price under mixed bundling
$P^* = 1.2356$; profit-maximizing post-merger bundle price under mixed bundling
$q^* = 0.5477$ and $q = 0.1765$; high-quality component quantities demanded at p^* and p
$Q^* = 0.6468$; bundle quantity demanded at $P^* = 1$
$P_{K\kappa} = 1.3956$; bundle price line that intersects Ω at points K and κ
$Q_{K\kappa} = 0.2068$; quantity demanded at $P_{K\kappa}$[22]

21 The function ψ is the inverse of θ and Ψ is the inverse of Θ.
22 Calculated as the solution to α in $\alpha - 0.8122 = 0.5834$ where (0.5834, 0.8122) are the coordinates of point K on Ω, and $Q_{K\kappa} = \Theta(P_{K\kappa})$.

$q_K = \theta(0.8122) = 0.3965$ where 0.8122 is the greater of the two values (i.e. coordinates) that define each point K and κ on Ω; component quantity demanded at $p = 0.8122$

$q_A = \theta(0.7583) = 0.4521$ where 0.7583 is the greater of the two values (i.e. coordinates) that define each point A and B on Ω; component quantity demanded at $p = 0.7583$

$q_{HH} = (2q^* - 1)/2 = 0.0477$; component sales to consumers who purchase both high-quality components pre-merger.

Table 2: Pre-merger vs. post-merger consumer surplus (CS) by purchased system type and the direction of the change in CS

Variable	Formula	Value
Hybrid system pre-merger $\rightarrow$ Hybrid system post-merger; CS decrease		
Mass	$2q$	0.353
Pre-merger CS	$2\int_0^q (\psi(q) - p^*)dq$	0.118
Post-merger CS	$2\int_0^q (\psi(q) - p)dq$	0.009
Change in CS		−0.109
Hybrid system pre-merger $\rightarrow$ High-quality system post-merger; CS decrease		
Mass	$Q^* - Q_{K\kappa} = 2(q_K - q)$	0.440
Pre-merger CS	$2\int_q^{q_K} (\psi(q) - p^*)dq$	0.108
Post-merger CS	$\int_{Q_{K\kappa}}^{Q^*} (\Psi(Q) - P^*)dQ$	0.041
Change in CS		−0.067
Hybrid system pre-merger $\rightarrow$ High-quality system post-merger; CS increase		
Mass	$Q_{K\kappa} - 2q_{HH} = 2(q_A - q_K)$	0.111
Pre-merger CS	$2\int_{q_K}^{q_A} (\psi(q) - p^*)dq$	0.014
Post-merger CS	$\int_{2q_{HH}}^{Q_{K\kappa}} (\Psi(Q) - P^*)dQ$	0.019
Change in CS		$+0.005$
High-quality system pre-merger $\rightarrow$ High-quality system post-merger; CS increase		
Mass	$2q_{HH}$	0.095
Pre-merger CS	$2\int_{q_{HH}}^{q^*} (\psi(q) - p^*)dq$	0.010
Post-merger CS	$\int_0^{2q_{HH}} (\Psi(Q) - P^*)dQ$	0.017
Change in CS		$+0.007$
Across all consumers		
Mass		1.000
Pre-merger CS		0.250
Post-merger CS		0.086
Change in CS		−0.164

Note: Totals may not add due to rounding.

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